XI
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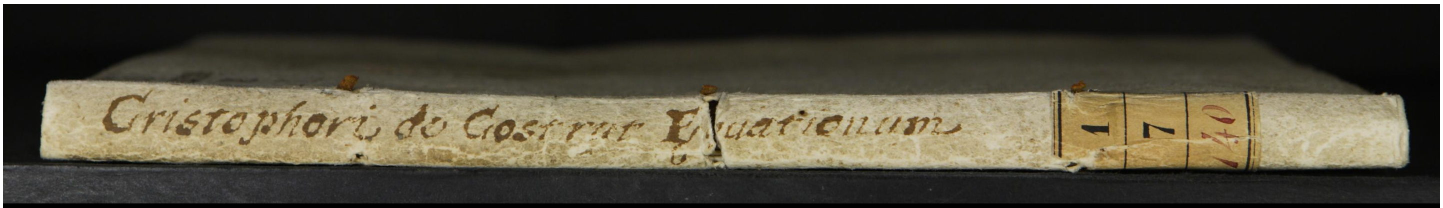
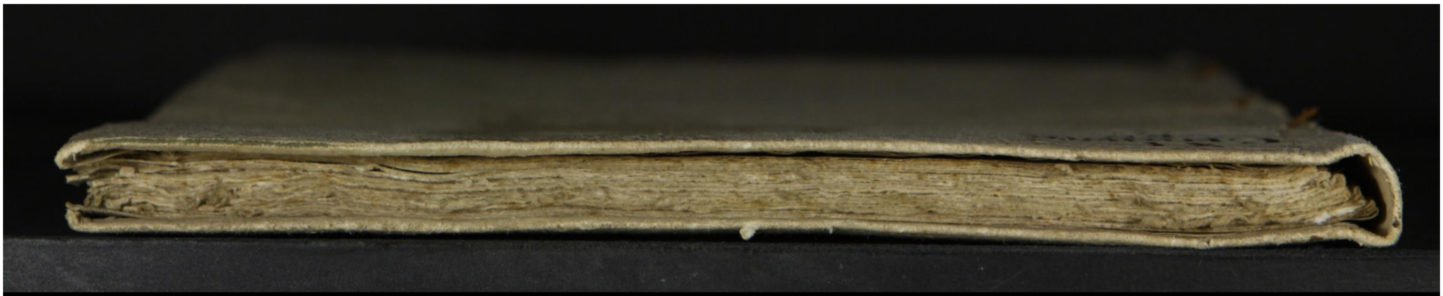


Cristophori de Cosrrur Equationum



XI
CHRISTOPH

DE CONSTRUCTIONE
AEQUATIONUM.

DE CONSTRUCTIONE ÆQUATIONUM.

HYACINTHI
CHRISTOPHORI.

J. C. Neapolitani.

DE CONSTRUCTIONE
ÆQUATIONUM.

Libellus.

NEAPOLI;
Ex Typographia Joſephi Roſelli. M. DCC.

Superiorum licentia.

Illuſtriſs. Viro literarum omnium ornatiſſimo

D. FRANCISCO MARCIANO

Supremì Italiæ Senatus apud Catholicum Regem
Regenti meritiſſimo.

HYACINTHUS CHRISTOPHORUS S. P. D.

DUm in vivis agebat Bernardus Pater meus, plurimorum Virorum Illuſtrium hujus Civitatis, ac præſertim Senatorii Ordinis, amicitiam ſibi comparavit; eorum autem, qui magis illam coluerunt, & amplexati ſunt, unus præ cæteris tu fuiſti Illuſtriſſimè, & Nobiliſſimè Vir, qui nedùm familiariter cum eo longo tempore egiſti, dum recta, ac firma valetu-

dine utebatur, & plurima in ipſum beneficia contuliſti, verùm, dum adverſa conflictaretur, minimè amoris, & benevolentiæ erga illum oblitus, pluries domi ægrotantem inviſiſti, multam diei partem doctо ſermone trahens; Præterea, poſtquam ultimum clauſerat diem, non deſtitiſti, ubi res tulerat, ejus memoriam honorificis encomiis cohoneſtare, & memor propenſæ voluntatis, & benevolentiæ, quam in ejus Patrem geſſerunt Regentes integerrimi Marcellus, & Io: Franciſcus Marcianus Pater, & Avus tuus, Viri Iuriſprudentiæ ſtudio, & literis omnibus inſignes, libenti, gratoque animo nos excepiſti, noſtriſq; rebus ſubvẽturum pollicitus, cum te chariſſimi Patris interitu, mærentes adivimus. Tot, tantisque igitur beneficiis devinctus, cum tibi eas, quàs optarem vices, rependere nequeam, in animi mei, ac obſervantiæ teſtimonium, opuſculum hoc *De Conſtructione Æquationum*, in quo explicatur me-

tho-

thodus conſtruendi problemata omnia, duarum curvarum interſectione ſecundum proprias ſedes, cujuſcunque generis ea ſint (res apud veteres , & recentiores Mathematicos diù deſiderata, ut teſtis eſt Pappus , & Carteſius) tuo nomini inſcriptum, in lucem prodire , & in hoc Patris mei vota adimplere volui , cui ſi ſors dediſſet publicæ luci exponere Vitas Illuſtrium Virorum,qui cum Io: Ioviano Pontano Neapoli floruerunt , unà cum Tractatu *de Ortu* , *& Progreſſu Academiæ* , non alii, quàm tuo nomini ſacraſſet . Cæterum , prætermiſſis officiis , quibus me, Patremque meum maximòpere demeritus es, ſi aliquis mihi deligendus erat, cui eſſet familiæ ſplendor,& nobilitas,ſumma dignitas , humanitas, & his omnibus præſtantior virtus , nemo erat quidem futurùs, ut delectus etiam inter paucos haberetur , quem tibi in hac laudis contentione , conferrem ; Nam tu clariſſimis ortus natalibus , præter Iuriſprudentiæ ſtu-

dium,

dium, quod te evexit ad tàm clarum, & conſpicuum munus, tanta es humanitate, ac virtute præditus, tantoque amore, & propenſione erga mathematicas, philoſophicaſque diſciplinas teneris, ut nemo ſit, quin de te magnificè ſentiat, ac loquatur. Supereſt jàm, ut hilari vultu, munuſculum hoc, & ſi tuis meritis impar, ſaltem uti non dubium mei in te amoris, ac obſervantiæ pignus excipias, atque potius ſpectes quod efficere optaſſem in promerenda tua gratia, quàm id, quod aſſequutus ſim. Vale, & Deus ad Patriæ bonum te diù incolumem ſervet.

EMINENTISS., E REVERENDISS. SIG.

IL Dottor Giacinto di Criſtofaro, ſupplicando eſpone a V. Em., come deſidera dare alle ſtampe un' Operetta matematica intitolata *De Conſtructione Æquationum Libellus*, La ſupplica per tanto di commettere la cenſura di eſſa a chi meglio a quella parerà, e l'haverà a grazia. ut Deus.

Reverendus Pater Nicolaus Parthenius Giannettaſius Soc. Jeſu, videat, & in ſcriptis referat, die 12. *Decembris* 1698.

JO: A. SILIQUINUS VIC. GEN.

D. Ianuarius de Auria Can. Deput.

EMINENTISS., E REVERENDISS. SIG.

LEgi librum, cui titulus eſt *De Conſtructione Æquationum Libellus*, in quo nihil animadverti, quod vel Fidei, vel moribus diſſonum ſit: imò auctum opus, & utile, quodque è Republica ſit in lucem prodire. Die 6. Martii 1699.

Emin. Tuæ Rev. Addictiſsimus Famulus

Nicolaus Parthenius Giannettaſius Soc. Jeſu.

Attenta ſupraſcripta relatione Reverendi Patris Reviſoris, quod poteſt imprimi, Imprimatur, die 14. *Maii* 1699.

JO: A. SILIQUINUS VIC. GEN.

D. Ianuarius de Auria Can. Dep.

ECCELLENTISS. SIGNORE.

IL Dottor Giacinto di Criſtofaro, ſupplicando eſpone a V. Ecc., come deſidera dare alle ſtampe un' Operetta matematica intitolata *De Conſtructione Æquationum Libellus*, La ſupplica per tanto di commettere la cenſura di eſſa a chi meglio a quella parerà, e l'haverà a gratia. ut Deus.

V. I. D. Joſeph Lucina videat, & in ſcriptis referat.

GASCON R. ANDREAS R. ANDREASSI R.
GVERRERO R. MERCADO R.

Proviſum per S.E. Neap. 7. Aprilis 1699. *Maſtellonus.*

ECCELLENTISS. SIGNORE.

PEr ubidire a' comandamenti di V. Ecc. hò letto il libro intitolato *De Conſtructione Æquationum Libellus*, e in quello non hò incontrato nulla, che ſia contra la Regal Giuriſdizione, e perciò ſi potrà dare alle ſtampe.

Di V. Ecc. Umiliſs., e Divotiſs. Servidore.

Giuſeppe Lucina.

Viſa ſupradicta relatione imprimatur, & in publicatione ſervetur Regia Pragmatica.

GASCON R. ANDREAS R. ANDREASSI R.
GVERRERO R. MERCADO R.

Proviſum per S.E. Neap. 13. Maii 1699. *Maſtellonus.*

Pag.	lin.	pro	lege
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83	11	propoſiræ	propoſitæ
84	17	coercetur	coercentur

HYACINTHUS CHRISTOPHORUS

Lectori.

Quibus ſuaſus rationibus, præter animi mei ſententiam, ad hæc ſcribenda me accinxerim, explicare non importunum amice Lector putavi. Bernardus Chriſtophorus pater meus, vir planè eruditus, multa ingenii ſui monimenta typis mandare parabat, quæ concinnaverat per otium, quod nanciſci potuit inter domeſticas curas, & irrequietum Juriſprudentiæ exercitium, cui non contemnenda nominis celebritate vacavit; inter ea erat ad Patriæ, & nominis ſui ornamentum opus illud diù deſideratum

Academia Pontani
ſive
Vitæ Illuſtrium Virorum, qui cum
Jo: Joviano Pontano Neapoli
Floruere.

In eo celebribus factis refertæ tradebantur vitæ Academicorum, non tàm oratorio, quàm hiſtorico genere perſcriptæ, ſcilicet Antonii Panormitæ, qui pri-

mus Neapoli conventum literatorum inſtituit, Joannis Joviani Pontani, ex quo nomen Academia deſumpſit, Actii Synceri Sannazarii, Ægidii S. R. E. Cardinalis Viterbienſis, Gabrielis Altilii, Petri Gravinæ, Alexandri ab Alexandro, Scipionis Capycii, Franciſci Ælii Marchesii, Hieronymi Borgiæ, Hieronymi Carbonis, Andreæ Matthæi Aquivivi Hadrienſium Ducis, Trajani Cabanilii Montellæ Comitis, Petri Gulini, ſive Compatris, Petri Summontii, Michaelis Marulli Byzantini, Puccii Florentini, Bartholomæi Scalæ, Jani Anysii, Joannis Pardi, Joannis Cottæ, & Henrici Puderici, additis Elogiis ad inſtar illorum Pauli Jovii, de Nicolao V. Romano Pontifice, & Alphonſo I. Aragoneo Neapolitanorũ Rege, qui poſtquam Barbaries inceſſerat Italiam, Romæ, & Neapoli politiores literas foverunt, & amplexati ſunt. Ad hoc opus veluti Iſagoge quædam attexebatur Tractatus de Ortu, & Progreſſu Academiæ, quo Author cũplectebatur diſciplinarum ſucceſſionem, ſumpto initio ab ea Cadmi ætate, qua literæ ex Ægypto in Græciam translatæ fuerunt. Argumentum ipſius erat, literas, liberaleſque artes, non ſecus, ac Imperia, & Reſpublicas, varias Orbis partes peragraſſe, ſuum fuiſſe initium ſortitas, incrementum, declinationem, oblivionem, & rerum converſione, reſtaurationem, juxtà Platonis ſententiam

αἰὼν πάντα φέρει, δολιχῶς χρῶνος ἤδεν ἀμίβειν
ὄνομα, καὶ μορφὰς, καὶ φύσιν, ἠδὲ τύχην.

Ævum cuncta fert, & longum tempus novit mutare
Nomina, facies, naturam, & fortunam rerum.

Oſtendebat Cadmum, & cæteros, qui primi ex Ægypto in Græciam literas adduxerunt, earum ope, Principatum fuiſſe adeptos, literarumq; doctrinam, ceu arcanum abſcondidiſſe, eam tantummodo illis aperientes, qui Regnum eſſent occupaturi; atque ita longo ævo bonas artes fuiſſe per manus traditas, & Græciam non ſolum ἱερογλυφικὰς, *ſive ſacras Ægyptiorum literas noviſſe, ſed & fabularem illam hiſtoriam, & involucra, veritatẽ apud Populos obſcurantia, in quibus occultabantur dogmata, vel ad Politiã, & Rem Sacrã, aut ad ſcientias, & mores pertinentia, quæ ſolis Principibus revelabantur. Addebat eos, qui interiores, & reconditas literas profitebantur, ad inſtar Sacerdotum Ægyptiorum, tanto in pretio apud Principes fuiſſe, ut ſocios, & conſortes Imperii illos facerent, & ſe in eorum Collegium cooptari non dedignarentur; Hos autem ſophos, ſive ſapientes vocatos, ea quæ primum poſteris tradiderunt dogmata, verſu ſcripſiſſe conſtat, quæ vel Deorum cultũ, & genus, vel Principũ laudes, aut Vitæ humanæ inſtitutum, & naturalium rerum cognitionem complectebantur, & hujuſmodi fuiſſe opera ſeptem Græciæ ſapientum, Solonis, Thaletis, Pittaci, Biantis, Chilonis Lacedemonii, Cleobuli, Periandri Corinthii, & aliorum. Explicabat poſtea, quâ viâ, vulgatâ literarum notitiâ, Primores cœperunt ſectas inſtituere, diſciplinarumque ſcientiam tradere, & primus omnium Pitagoras Italicæ Sectæ conditor, relicto ſophi nomine, uti nimis elato, philoſophus, ideſt ſapientiæ amicus appellari voluit, ex quo poſtmodum literarum doctrinam profitentes, Philoſophi vocati fuerunt, & Sapientiæ ſtudium Philoſophia; Enarrabat ſimiliter, quo pacto ex Philoſophia Poëſis fuerit diviſa, & Poëtæ, qui Philoſophi*

erant

erant, peculiare nomen aſſumpſerint; ſicque diſſerebat de origine Poëmatis Heroici, Tragediæ, Comediæ, & variis apud Græcos Poëſeos generibus, eorumque inventoribus; Ad hæc, qua ratione ex Pherecide Athenienſi, qui primus (ex Porphirio apud Suidam) ſoluta oratione ſcribere cæpit, ſuccedentibus temporibus, Rhetores, ſive Oratores, I. Conſulti, Hiſtorici, Dialectici, Onomaſtici, ſive Critici, & Grammatici inſurrexerunt, & quemadmodum à Philoſophia fuerint diviſæ, Medicina, Aſtronomia, Geometria, Arithmetica, & cætera liberales diſciplinæ, quarum progreſſum ſuis locis aperiebat; Præterea ludorum Gymnicorum originem illuſtrabat, aſſerens Authorem illorum Minoa, qui tenuit poſt Rhadamantum Regnum Cretenſe, ut juventutem ad bellicos labores exerceret; & latè diſſerebat de Olympicis, Pythiis, Iſthmicis, Nemeiſque ludis, & celeberrimis Græciæ Gymnaſiis, Lyceo, Cynoſarge, & Academia, in quibus juvenes exercebantur; Continuato deinde ordine, loquebatur de conditionibus juvenum, qui in ludorum exercitium admitti debebant, de variis ipſorum generibus, regulis, & inſtitutionibus, quas Gymnaſiarchæ, ſive Gymnaſiorum præfecti ſeſe exercentibus tradere ſolebant, & demum de præmiis, quibus victores donabantur, & quia in Olympicis erant corona ex Bacchare, ex Oleaſtro in Pythiis, Pino in Iſthmicis & Apio in Nemeis, hac arrepta occaſione, de Coronis, earum uſu, & ſignificatu nonulla cumulabat. Addebat hæc Juvenum collegia, Ephebiaca appellata, Mamaluccorum *collegijs apud Ægyptios,* & Janizzerorum *apud Turcas ſimilia fuiſſe, verum non omnibus Civitatibus licebat illa habere, ſed liberis tantùm, & Principibus viris, aliorum legibus*

ſolu-

ſolutis, & ita obſcurũ Strabonis locum de noſtra Neapoli obiter enodabat, quæ inter alia Græcorum Inſtitutorũ veſtigia, ſervabat Gymnaſium, Ephebiacum, Curias, quas Græci phratrias vocabant, & Græca nomina Romanis impoſita, ut ex ejus verbis, colligi poteſt

πλεῖστα δ' ἴχνη τῆς Ελλωικῆς ἀγωγῆς ἐνταῦθα σώζεται, γυμνάσιά τε, καὶ ἐφηβιακὰ, φρατρία, καὶ ὀνόματα Ελληνικὰ, καὶ περόντα Ῥωμαίων. *Lib. 5. Geogr.*

Unde deducebat liberam fuiſſe Civitatem, etiam in ipſis Strabonis temporibus, qui ſub Tiberio floruit; Narrabat poſtmodum, Peloponeſſiaco bello, Regionibus devaſtatis, & cæſis juvenibus, militaribus in exercitiis educatis, hæc loca diù deſerta, philoſophos occupaviſſe, qui licet sũmo eſſent in pretio apud Græcos, nullum tamen locum publicum habebant, ſed propriis in ædibus diſcipulos excipiebant, docebantque philoſophiam; Et ita juxtà propoſitum ſibi argumentum, quæ pro militaribus prius diſciplinis loca inſtituta fuerant, mutato rerum ordine, literis ceſſiſſe oſtendebat, nominaque illa, Gymnaſium, Lyceum, & Academiã ſcholas literarias retinuiſſe, & verba illa, Deſcendere in arenam, Conſerere manus, *& ſimilia, ad corporis exercitamenta pertinentia, pro ſcientificis velitationibus uſurpata fuiſſe. Referebat in Lyceo primum docuiſſe Archelaum, Anaximenis auditorem, è Thaletis Mileſii ſecta, quæ ſummòpere in Græcia vigebat, & poſtea ejus diſcipulum Socratem; In Cynoſarge Antiſtenem Cinicæ ſectæ authorem, & in Academia Platonem, vacuo relicto Lyceo, ceu loco infauſto, ob mortem præceptoris Socratis, qui calumniis Anyti, Melyti, & Lyconis circumventus, undecim Virorum judicio, capitalem ſententiam paſſus fuit; at ibi aliquanto poſt docuiſſe teſtabatur*

Ari-

Ariſtotelem Stagiritem, Platonis diſcipulum, ex quo peripathetici inſurrexerunt, ſic vocati, quia inambulantes in Lyceo diſputabant; Continuato deinde per varias Orbis partes Academiæ proceſſu, Stoicam philoſophiam ab Zenone Cittico originem accepiſſe, qui Cinicam ſectam deteſtatus, in porticu Pæcile dicto, celebri Urbis loco, ϛοὰ à Græcis nuncupato, ſcholam aperuit, & Epicuream ab Epicuro, qui ex Italica philoſophia prodiens, primus Athenis in hortis docuit. Sic igitur ordinem reſumens, aſſerebat numeroſiſſimos redditos philoſophos, varias in ſectas diviſos, & philoſophiam per omnes gentes longè, latèque diffuſam; At Principes, qui jàm armis Imperium firmaverant, literas odere, & Philoſophos geſtorum ſuorum judices ægrè ferebant; itaque moleſti, atque inviſi apud Magiſtratum eſſe cæperunt, & nonnulli eorum jejunis, frigidiſque prætextis, indicta cauſa, damnati, ſicuti Anaſſagoras, Socrates, aliique, & loca illa pluries disjecta, & deſerta remanſerunt. Interea explanabat, qua ratione, aucto Romano Imperio, literæ Romam ex Græcia fuerint translatæ, & Poëſis, Juriſprudentia, Rhetorica, Hiſtoria, Philoſophia, cæteræque liberales diſciplinæ illùc migraverint; Quis primùm Romæ ſcholam aperuerit, ac docere cæperit, & quomodo hæ diſciplinæ ad ſummum perfectionis gradum pervenerint; Simul, reſumpto Academiæ ordine, aſſerebat primam in Italia fuiſſe illam, quam Cicero in ſua villa propè Puteolos inſtituit, ubi Tuſculanas, & Academicas quæſtiones compoſuit, quæ Academia poſtea intermiſſa fuit, interempto ejus Inſtitutore trium Principum proſcriptione, etenim præfata villa primò devenit ad Aſinium Gallum, deinde ad Aruntium Stel-

lam,

lam, qui licet literis inſignes, nullam tamen Academiæ curam habuerunt, alteram verò illam, quam Romæ ſub Galieno Principe Plotinus philoſophus Platonicus ordinavit. Exponebat deinceps, quomodo labente Romano Imperio, literæ, ac liberales diſciplinæ paulatim minui cæperunt, & demum ex irruptione Barbarorum ab Septentrionali plaga, eo collapſo, & cunctis ferro, flammiſque vaſtatis, penitus everſæ, ac ſepultæ remanſerunt; Narrabat poſtea, mutato rerum Statu, literas revixiſſe, & parvula ipſarum lumina in noſtra Regione apud Caſſinates Monachos primùm apparuiſſe, qui diſciplinarum fragmenta, veluti magni naufragii tabulas, ſervarunt, quò ſacrarum literarum ſtudia aptius excolerent; Poſthac, tempore Lotharii Cæſaris, reperto Melphiæ apud Lucanos Juris Civilis corpore, Juriſprudentiæque ſcholis apertis Bononiæ, Patavii, atque in Gallia, & ætate Friderici II. Imperatoris, Neapoli, iterum hoc ſtudium in variis Europæ partibus florere cæpiſſe. Indicabat ſimul, quo pacto, cura, & ſtudio ejuſdem Friderici Cæſaris, qui rem literariam magno proſequebatur amore, opera Ariſtotelis phyſica, quæ Mauri in Hiſpaniam advexerant, in latinum idioma fuerint converſa ex arabo, in quod illi unà cum Aſtrologia, & Galeni Medicina, quam ſolam receperant, è græco tranſtulerant, & hæc aditum poſtea ad Logicam, Dialecticam, & Theologicas quæſtiones apud Cucullatos aperuerit, qui per omnes ſcholas Ariſtotelis philoſophiam propagarunt, illique inhæſerunt Medici, Galeni opinioni addicti; Poſtmodum recenſens renaſcentium literarum incrementum, explicabat qua ratione Emanuel Criſoloras orator miſſus in Italiam ad Catholicos Principes ab Imperatore Coſtantino-

politano Emanuele Paleologo, ut auxilium ab illis peteret contrà Bajazetem Turcarum Principem, incolatum ibi firmaverit, & Græcas literas, ſeptingentorum ſpatio annorum ab ea exulantes, reſtituerit, advectis antiquorum Authorum libris, & Venetiis, Florentiæ, Romæ, Ticini, ac Mediolani ſcholis inſtitutis, è quibus Viri omni ævo celebres prodiere; Præterea, quemadmodum Nicolaus V. ad Summum Pontificatum evectus, & Alphonſus I. Aragoneus Neapolitanorum Rex, politiores literas Romæ, & Neapoli reſtaurarunt, memorans inſignem Romæ ſcholam, in qua Laurentius Valla Romanus docuit, & Julius ille Sanſeverinus, qui legibus propriis, ac particularibus inſtitutis, illuſtrem ibi erexit Academiam, quæ viros ingenio, & literis ſanè mirabiles extulit; Deinde percenſebat nobilem illam Porticum, Antonianam vocatam ab Antonio Panormita Conſiliario, & intimo Alphonſi Regis, qui primus Neapoli congreſſum literatorum inſtituit, in quo ſelectiores de rebus literariis Stoicorũ more diſſerere ſolebant, & hi velut in φρατρίαν redacti, proprio nomini aliquid addebant, vel immutabant, ita ut Antonius ipſe, qui de Bononia cognominabatur, Panormita appellari voluit, & Pontanus, qui prius Joannes vocabatur, Jovianus; Denique cõmemorabat omni gloria majorem illã Academiam, Pontani dictam, quam poſt Panormitæ obitum, Pontanus alumnus, non longè ab ipſius Porticu, propè Regionem Montaneæ in propria domo ordinavit, in qua literarum ſtudiis dediti, aliorum veſtigiis inhærentes in nominum, cognominumque mutatione, de rebus ſcientificis Academicorũ more diſſerebant, & hanc primã fuiſſe teſtabatur poſt Barbariem ad inſtar illius Platonis, ex qua literati viri, velut ex equo

Troja-

Trojano exeuntes ; undique bonas Artes propagarunt , adeout per eam Poëſis , Rethorica , Hiſtoria, Juriſprudentia , Grammatica, Critica , & deinceps Philoſophia, Medicina, Geometria, Aſtronomia, cæteræque liberales diſciplinæ,perfectiores, & cultiores evaſerint ; Rurſus oſtendebat,qualis fuerit, & ad præſens ſcientiarum Status in Orbe reperiatur,qui publici literatorum Conventus erecti , & adhuc, quæ ſcholæ, & Academiæ in Italia, Gallia, Anglia, Hollandia, Alemannia, Hiſpania, cæteriſque Orbis partibus ad cultum literarum promovendum ſint inſtitutæ; Ad ſummam,ne te pluribus morer, perſtringebat ab Cadmi initio uſque ad præſentem ætatem, ordine quidem hiſtorico , non autem Criticorum more , progreſſum literarum , ac diſciplinarum omnium , repetens ex omni memoria,quæ Doctrinæ & Artes,quibus Mundi ætatibus,& Regionibus floruerint,earum antiquitates, & peragrationes per diverſas Orbis partes, rurſus declinationes, obliviones , & inſtaurationes , aperiens in ſingulis artibus inventionis occaſionem, & originem, tradendi morem, & diſciplinam , ſectas, & controverſias maximè celebres, quæ homines doctos tenuerunt , calumnias, quibus patuerunt , laudes , & honores, quibus decoratæ ſunt , Authores præcipuos, libros præſtantiores ,ſcholas ,ſucceſſiones , Academias, Societates, Collegia, Ordines, & demum omnia, quæ ad Statum literarum ſpectant,& Bacconus de Verulamio in Hiſtoria literarum deſiderabat. De Augm. Scientiar. lib. 2. c. 4. *Opus igitur hoc tàm utile , & eruditum , cum è ſuo Muſeo ablatum fuiſſet eadem die,qua ex hac vita , lugentibus filiis, & familiaribus coram eo adſtantibus, diſceſſit , amplius recuperari non potuit magna literarum, Patriæ, & nominis ſui jactura ; Propte-*

rea ſucceſſu tàm deplorando admonitus, non benè proprii nominis gloriæ conſulere eos novi, qui ſua opera publicæ luci exponere nimis lentè adnituntur; illa enim facilè oblivione obruuntur, vel caſibus, quibus adhuc ſuperſtites Authores ſubjacere ſolent, vel tandem incuria hæredum, qui bonorum magis, quàm gloriæ majorum eſſe ſolent avidi; præter animi propoſitum, hoc opuſculum, ut typis ederetur, permiſi, ſuadentibus etiam eruditiſſimis amicis Nicolao Galitia, & Joſepho Lucina, quibus morem gerere non Tom. 3. Epiſt. 48. *recuſavi, ex eo præſertim, quòd Carteſius idem à ſui temporis Mathematicis ad Scientiæ augmentum, faciendum, dicebat*. Operam dare deberent componendi omnia loca superſolida, ut ego compoſui ſolida, & denique conſtruendi omnia problemata, quæ adſcendunt ad quadrum quadri-quadrati, aut ad cubum-cubi.

Vale, & vota tua cumulatè impleat Deus.

DE CONSTRUCTIONE ÆQUATIONUM.

Subtiliſſimæ, certiſſimæq; ſcientiarum, qualis eſt Analyſeos doctrina, id præcipuum incommodi accidit, quod ſublimioris tantùm ingenii Viris penitùs perſpecta, plenèque cognita eſſe conſuevit: hi enim aliorum inventis neglectis, propria ſolum excolere, atque in lucem proferre dignantur. Qui autem facilioribus ſunt moribus, & explanandis jam traditis diſciplinis duntaxat ſe ſe dedunt, non admodum excelſa pollere ſolent mente, ut illa præſtare valeant, quæ huic ſcientiæ deeſſe comprehenduntur. Unde factum eſt, quod rarò, aut nunquam veterum inventis aliquid novi addant, ut paſſim videre eſt apud eos, qui Encyclopædias, aut Curſus diſciplinarum componunt; in illis enim ſilentur, quæ majoris ſunt ponderis, aut profundiorem eruditionem requirunt, leviori-

bus ſolummodo explicatis; Itaque partem illam, quæ conſtructionem omnis generis æquationum unicâ, & ſimpliciſſimâ viâ oſtendit, ab aliis interruptâ, atque incoſtanti methodo indicatam, nemo eorum, qui mentis acumine reliquis præſtant, altiùs promovere, & ad perfectiorem ordinem redigere cogitavit; Quare mihi relictam hanc partem videns, unà coronidem huic ſcientiæ imponere, atque illius ſtudioſis prodeſſe tentavi, vagâ illâ conſtruendi ratione, modò per cuborum latera, modò per lineas circulis inſcriptas, poſtpoſitâ, eam in medium afferre viſum eſt, quæ per ſimpliciſſimarum curvarum linearum interſectionem haberi poteſt, cujuſcunque generis ſint problemata: Bonorum, atque intelligentium judicio rem non contemnendam me præſtitiſſe ſpero, quam ſi his placuiſſe cognovero, ad ſimilia, & fortè etiam meliora animum deinceps appellere non gravabor.

Quoniam verò hæc methodus uſu faciliùs, quam præceptis percipitur, longo ſermone Lectores non morabor, ſed ad exempla progrediar.

Proponatur invenienda conſtructio æquationis quatuor dimenſionum $x^4 - px^3 - qx^2 - rx - s \propto o$, ſivè $x^4 \propto px^3 + qx^2 + rx + s$, in qua omnes extant termini.

Hæc ex ſui natura quatuor explicari poteſt radicibus, tribus nimirum falſis, & quarta vera, ſed majori, ut ſignum $-px^3$ indicat, omnibus falſis ſimul ſumptis, vel duabus tantùm radicibus, majori ſcilicet vera, & minori falſa, cum reliquæ duæ imaginariæ eſse poſſint. Quomodo autem imaginariæ radices in quacunque æquatione ſint dignoſcen-

dæ,

dæ, alibi ſi otium ſuppetet, oſtendemus.

Methodus pro conſtructione invenienda ita procedit.

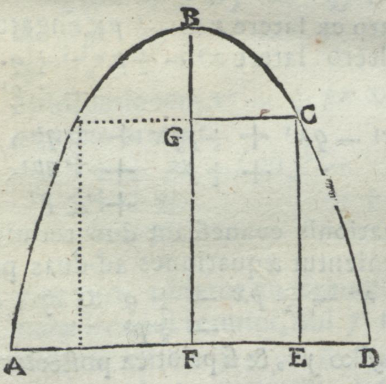
Formetur quadratum ex latere $x^2 - \frac{1}{2}px$, ſcilicet $x^4 - px^3 + \frac{1}{4}p^2x^2$.

Addatur utrique parti æquationis portio $-px^3 + \frac{1}{4}p^2x^2$, & habebitur hæc altera æquatio

$$x^4 - px^3 + \frac{1}{4}p^2x^2 \infty qx^2 + rx + s$$
$$+ \frac{1}{4}p^2$$

Conſtituatur plano-planum a^2y^2 æquale utrique ejus parti, ſumpta a, facilitatis gratia, pro unitate, & duæ emergent æquationes; Altera nimirum $x^4 - px^3 + \frac{1}{4}p^2x^2 \infty a^2y^2$, ſive $x^2 - \frac{1}{2}px \infty ay$ ad parabolam, cujus parameter a, ſive unitas, CE axi parallela $\infty\ y$. CG, ſive EF quantitas, qua CE ab axe diſtat $\infty\ \frac{1}{4}p$. AE major baſis pars $\infty\ x$, & DE minor $\infty\ x - \frac{1}{2}p$, quod ita demonſtratur.

Pappus in fine lib. 4.



Quia rectangulum ſub axe, ejuſve parallelis, & parametro æquale eſt rectangulo ſub baſis ſegmentis, nimirum rectangulum ex a, & CE æquale rectangulo ex AE in ED, poſitis iiſdem ſpeciebus, habebitur æquatio inventa $x^2 - \frac{1}{2}px \infty ay$. Q. E. D.

Hinc colligi poteſt quàm oſcitanter Chriſtopho-

Lib. 2. ſect. 2. cap. 1. prop. 6. rus Sturmius in ſua Matheſi enucleata aſſeruerit hanc parabolæ proprietatem fuiſse veteribus Mathematicis incognitam, & Thomam Strode ex ingenioſo M. S. de ſectionibus conicis excerptam *loc. cit.* Thomæ Backero cõmunicaſse, cum apud Pappum ea reperiatur demonſtrata, & ab Sluſio in ſua Analyſi, Florimondo de Beaune in Adnotationibus ad Carteſii Geometriam, aliiſque Mathematicis, qui Backerum præceſserunt, pro vulgatiſſima habeatur.

Altera verò æquatio erit

$$\begin{array}{l} qx^2 + rx + ſ \propto a^2y^2, \text{ ſcilicet} \\ + \tfrac{1}{4}p^2 \end{array}$$

$$\begin{array}{l} aqx^2 + a^2rx + a^3ſ \propto a^2y^2, \text{ hoc eſt} \\ + \tfrac{1}{4}p^2 \end{array}$$

$$\begin{array}{l} \frac{qx^2}{a} + rx + aſ \propto y^2 \text{ ad hyperbolã.} \\ + \tfrac{1}{4}\frac{p^2}{a^2} \end{array}$$

At ſi pro quadrato ex latere $x^2 - \tfrac{1}{2}px$ fingatur quadratum ex altero latere $\begin{array}{l} x^2 - \tfrac{1}{2}px - \tfrac{1}{2}q \\ \qquad\qquad\quad - \tfrac{1}{8}p^2 \end{array}$,

$$\text{ſcilicèt } \begin{array}{llll} x^4 - px^3 - qx^2 & + \tfrac{1}{2}pqx & + \tfrac{1}{4}q^2 & , \\ & + \tfrac{1}{8}p^3 & + \tfrac{1}{8}qp^2 & \\ & & + \tfrac{1}{64}p^4 & \end{array}$$

ut propoſitæ æquationis evaneſcant duo termini $px^3 + qx^2$, invenientur æquationes ad duas parabolas, nimirum $\begin{array}{l} x^2 - \tfrac{1}{2}px - \tfrac{1}{2}q \propto ay, \\ \qquad\qquad\quad - \tfrac{1}{8}p^2 \end{array}$ &

$$\begin{array}{l} \qquad rx + aſ \propto y^2, \text{ \& ſi prior ex poſteriore,} \\ + \tfrac{1}{2}\frac{pq}{a} + \tfrac{1}{4}q^2 \\ + \tfrac{1}{8}\frac{p^3}{a^2} + \tfrac{1}{8}\frac{qp^2}{a} \\ \qquad + \tfrac{1}{64}\frac{p^4}{a^2} \end{array}$$

vel

vel poſterior ex priore auferatur, altera ipſarum ad circulum tranſibit; ac proindè parabolæ, & circuli beneficio propoſitæ æquationis conſtructio faciliter hac via habebitur.

Sed ut ex hyperbola ad circulum tranſeamus; Primo in locum x^2 ſubrogabimus ejus valorem $\frac{1}{2}px + ay$, & habebimus alteram æquationem

$$\begin{array}{llll} \frac{1}{2}\frac{qpx}{a} & + & qy + rx + a\text{ſ} \infty y^2, \\ + \frac{1}{8}\frac{p^3}{a^2} & + & \frac{1}{4}\frac{p^2}{a} \end{array}$$

ideſt

$$\begin{array}{llll} \frac{1}{2}\frac{qpx}{a} & + a\text{ſ} \infty y^2 & - & qy, \text{ ſive} \\ + \frac{1}{8}\frac{p^3}{a^2} & & - & \frac{1}{4}\frac{p^2}{a} \\ + r & & & \end{array}$$

$$\begin{array}{llll} \frac{1}{2}qpx & + \text{ſ} \infty y^2 & - & qy. \\ + \frac{1}{8}p^3 & & - & \frac{1}{4}p^2 \\ + r & & & \end{array}$$

Deinde auferemus ab ea æquationem $x^2 - \frac{1}{2}px \infty ay$, & ad circulum æquatio orietur.

$$\begin{array}{lllll} -x^2 & + \frac{1}{2}px & + \text{ſ} \infty y^2 & - & qy; \\ & + \frac{1}{2}qp & & - & \frac{1}{4}p^2 \\ & + \frac{1}{8}p^3 & & - & a \\ & + r & & & \end{array}$$

Si enim utrique ejus parti addatur quadratum dimidii quantitatis notæ ſecundi termini, ubi y^2 reperitur, ideſt quadratum ex $-\frac{1}{2}q - \frac{1}{8}p^2 - \frac{1}{2}$, vel ex $\frac{1}{2}q + \frac{1}{8}p^2 + \frac{1}{2}$, nimirum $\frac{1}{4}q^2 + \frac{1}{8}p^2q + \frac{1}{2}q + \frac{1}{64}p^4 + \frac{1}{8}p^2 + \frac{1}{4}$, habebitur æquatio

$-x^2$

$$\begin{array}{lllllll}
-x^2 & +\frac{1}{2}px & +\frac{1}{4}q^2 & \propto y^2 & -\; qy & +\frac{1}{4}q^2 & ; \\
 & +\frac{1}{2}qp & +\frac{1}{8}p^2q & & -\frac{1}{4}p^2 & +\frac{1}{8}p^2q & \\
 & +\frac{1}{8}p^3 & +\frac{1}{2}q & & =\; a & +\frac{1}{2}q & \\
 & +\; r & +\frac{1}{64}p^4 & & & +\frac{1}{64}p^4 & \\
 & & +\frac{1}{8}p^2 & & & +\frac{1}{8}p^2 & \\
 & & +\frac{1}{4} & & & +\frac{1}{4} & \\
 & & +\; \text{ſ} & & & &
\end{array}$$

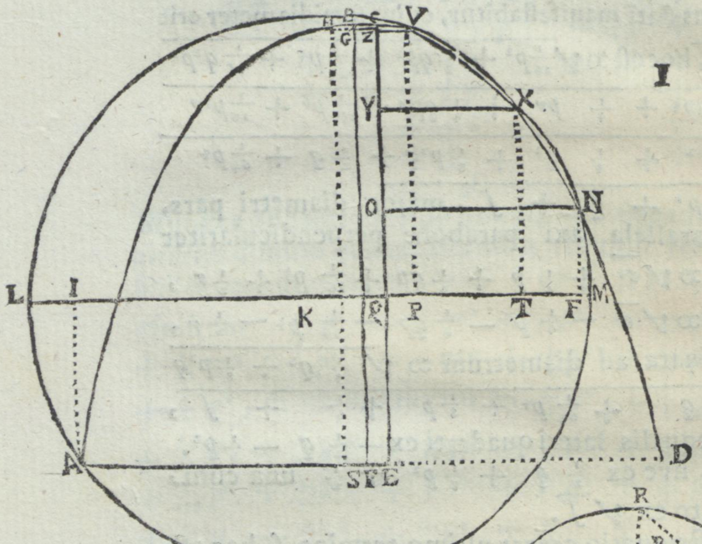
Si poſtea addatur parti, in qua eſt $-x^2$ quadratum alterius dimidii quantitatis notæ ſecundi termini, ſcilicet quadratum ex $\frac{1}{4}p + \frac{1}{4}qp + \frac{1}{16}p^3 + \frac{1}{2}r$, nimirum $\frac{1}{16}p^2 + \frac{1}{8}qp^2 + \frac{1}{32}p^4 + \frac{1}{16}q^2p^2 + \frac{1}{32}qp^4 + \frac{1}{4}pr + \frac{1}{4}qpr + \frac{1}{256}p^6 + \frac{1}{16}p^3r + \frac{1}{4}r^2$, & ex aggregato $\propto \frac{1}{16}p^2 + \frac{1}{8}qp^2 + \frac{1}{32}p^4 + \frac{1}{16}q^2p^2 + \frac{1}{32}qp^4 + \frac{1}{4}pr + \frac{1}{4}qpr + \frac{1}{256}p^6 + \frac{1}{16}p^3r + \frac{1}{4}r^2 + \frac{1}{4}q^2 + \frac{1}{8}p^2q + \frac{1}{2}q + \frac{1}{64}p^4 + \frac{1}{8}p^2 + \frac{1}{4} + \text{ſ}$, quod brevitatis cauſa ponitur $\propto e^2$, idem quadratum auferatur, ut ea mutetur in

$$\begin{array}{llll}
-x^2 & +\frac{1}{2}px & +\; e^2 & , \\
 & +\frac{1}{2}qp & -\frac{1}{16}p^2 & \\
 & +\frac{1}{8}p^3 & -\frac{1}{8}qp^2 & \\
 & +\; r & -\frac{1}{32}p^4 & \\
 & & -\frac{1}{16}q^2p^2 & \\
 & & -\frac{1}{32}qp^4 & \\
 & & -\frac{1}{4}pr & \\
 & & -\frac{1}{4}qpr & \\
 & & -\frac{1}{256}p^6 & \\
 & & -\frac{1}{16}p^3r & \\
 & & -\frac{1}{4}r^2 &
\end{array}$$

circulus ſtatī manifeſtabitur, cujus ſemidiameter erit $\infty \sqrt{e^2}$, hoc eſt $\infty \sqrt{\frac{1}{16}p^2 + \frac{1}{8}qp^2 + \frac{1}{32}p^4 + \frac{1}{16}q^2p^2 + \frac{1}{32}qp^4 + \frac{1}{4}pr + \frac{1}{4}qpr + \frac{1}{256}p^6 + \frac{1}{16}p^3r + \frac{1}{4}r^2 + \frac{1}{4}q^2 + \frac{1}{8}p^2q + \frac{1}{2}q + \frac{1}{64}p^4 + \frac{1}{8}p^2 + \frac{1}{4} + \text{ſ}}$, major diametri pars, cui parallela axi parabolæ perpendiculariter inſiſtit $\infty \sqrt{e^2} + \frac{1}{4}p + \frac{1}{4}qp + \frac{1}{16}p^3 + \frac{1}{2}r$, minor $\infty \sqrt{e^2} - \frac{1}{4}p - \frac{1}{4}qp - \frac{1}{16}p^3 - \frac{1}{2}r$, & ordinata ad diametrum $\infty \sqrt{\frac{1}{4}q^2 + \frac{1}{8}p^2q + \frac{1}{2}q + \frac{1}{64}p^4 + \frac{1}{8}p^2 + \frac{1}{4} + \text{ſ}}$, ideſt æqualis lateri quadrati ex $- \frac{1}{2}q - \frac{1}{8}p^2. - \frac{1}{2}$, ſive ex $\frac{1}{2}q + \frac{1}{8}p^2 + \frac{1}{2}$ unà cum quadrato ex $\sqrt{\text{ſ}}$.

Sed ſi æquatio careat ultimo termino ſ, hoc eſt trium ſit duntaxat dimenſionum, erit ea tantummodo $\infty \sqrt{\frac{1}{4}q^2 + \frac{1}{8}p^2q + \frac{1}{2}q + \frac{1}{64}p^4 + \frac{1}{8}p^2 + \frac{1}{4}}$, ſive $\infty \frac{1}{2}q + \frac{1}{8}p^2 + \frac{1}{2}$, ut quiſque facili negotio deprehendet.

Et hæc eſt Analyſis propoſitæ æquationis, ex qua mira facilitate oritur conſtructio.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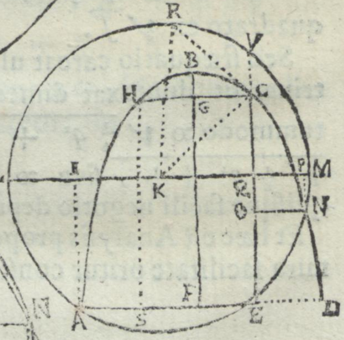
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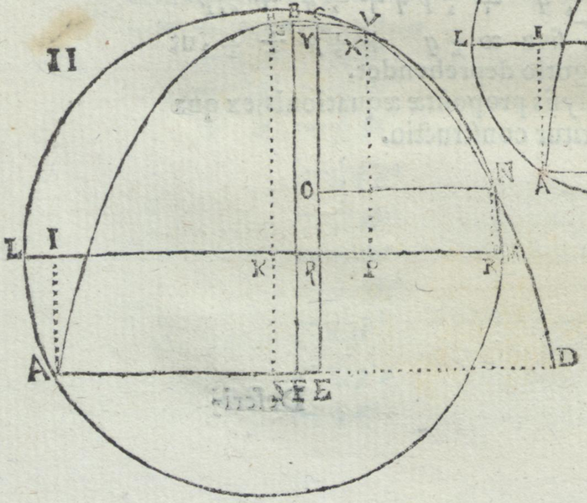
III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II

Deſcribatur parabola ABD, cujus parameter a, ſive unitas. Ducatur ex puncto C recta CGH $\infty\ \frac{1}{2}p$, vel CG ordinata ad axim $\infty\ \frac{1}{4}p$. Sit CE axi parabolæ parallela $\infty\ y$. Sumatur portio ipſius CQ $\infty\ \frac{1}{2}q + \frac{1}{8}p^2 + \frac{1}{2}$. Ducatur ex puncto Q ad rectos angulos recta QK $\infty\ \frac{1}{4}p + \frac{1}{4}qp + \frac{1}{16}p^3 + \frac{1}{2}r$, & producta in rectum ſumatur KL $\infty\ \sqrt{e^2}$, ideſt æqualis lateri quadrati, quod cõponũt quadrata ex KQ, QC, hoc eſt KC, & $\sqrt{ſ}$, ſive CR, Fig. III. nimirum $\infty\ \sqrt{\frac{1}{16}p^2 + \frac{1}{8}qp^2 + \frac{1}{32}p^4 + \frac{1}{16}q^2p^2 + \frac{1}{32}qp^4 + \frac{1}{4}pr + \frac{1}{4}qpr + \frac{1}{256}p^6 + \frac{1}{16}p^3r + \frac{1}{4}r^2 + \frac{1}{4}q^2 + \frac{1}{8}p^2q + \frac{1}{2}q + \frac{1}{64}p^4 + \frac{1}{8}p^2 + \frac{1}{4} + ſ}$. Statuatur hæc recta circuli ſemidiameter, cujus centrum K; dico hunc circulum eſſe quæſitum, qui verticem parabolæ includens, eam in quatuor punctis A, V, X, N ſecabit, Fig. I. vel in tribus A, V, N, aut in duobus A, & N, Fig. II. Fig. III. quod accidit, vel cum duæ ex falſis radicibus inter ſe æquantur (tunc enim punctum V convenit cum puncto X, & circulus ibi parabolam tangit), vel cum eæ imaginariæ exiſtunt.

Si igitur ex his punctis, in quibus parabolam circulus ſecat, orthogonales ducantur ad CE axi parallelam VZ, XY, NO, & AE; erunt hæ propoſitæ æquationis radices, AE nimirum vera, cæteræ autem falſæ, quod ita demonſtratur.

Quia AE inventa eſt $\infty\ x$, & CGH, ſive ES poſita $\infty\ \frac{1}{2}p$, erit AS, ſive ED $\infty\ x - \frac{1}{2}p$; cũque propter parabolam, rectangulum ex AE in ED ſit æquale

Pappus loc. cit. rectangulo ex parametro in rectam CE, ſcilicet $x^2 - \frac{1}{2}px \propto ay$, erit propterea y, ſive CE $\propto \frac{x^2}{a} - \frac{1}{2}\frac{px}{a}$, hoc eſt $\propto x^2 - \frac{1}{2}px$. Quia ſimiliter CQ poſita eſt $\propto \frac{1}{2}q + \frac{1}{8}p^2 + \frac{1}{2}$, dempta hac portione ex recta CE, ſcilicet ex $x^2 - \frac{1}{2}px$, reliqua erit QE, ſive AI $\propto x^2 - \frac{1}{2}px - \frac{1}{2}q - \frac{1}{8}p^2 - \frac{1}{2}$. Rurſus quia circuli ſemidiameter poſitus fuit ex conſtructione $\propto \sqrt{e^2}$, nimirum $\propto \sqrt{\frac{1}{16}p^2 + \frac{1}{8}qp^2 + \frac{1}{32}p^4 + \frac{1}{16}q^2p^2 + \frac{1}{32}qp^4 + \frac{1}{4}pr + \frac{1}{4}qpr + \frac{1}{256}p^6 + \frac{1}{16}p^3r + \frac{1}{4}r^2 + \frac{1}{4}q^2 + \frac{1}{8}p^2q + \frac{1}{2}q + \frac{1}{64}p^4 + \frac{1}{8}p^2 + \frac{1}{4} + \text{ſ}}$, & KQ $\propto \frac{1}{4}p + \frac{1}{4}qp + \frac{1}{16}p^3 + \frac{1}{2}r$, erit tota LQ major diametri portio $\propto \sqrt{e^2} + \frac{1}{4}p + \frac{1}{4}qp + \frac{1}{16}p^3 + \frac{1}{2}r$, ex qua ſi auferatur AE, ſive IQ $\propto x$, relinquetur LI $\propto \sqrt{e^2} + \frac{1}{4}p + \frac{1}{4}qp + \frac{1}{16}p^3 + \frac{1}{2}r - x$, ac proinde IM erit $\propto \sqrt{e^2} - \frac{1}{4}p - \frac{1}{4}qp - \frac{1}{16}p^3 - \frac{1}{2}r + x$. Sed propter circulum, rectangulum ex LI in IM, hoc eſt ex $\sqrt{e^2} + \frac{1}{4}p + \frac{1}{4}qp + \frac{1}{16}p^3 + \frac{1}{2}r - x$ in $\sqrt{e^2} - \frac{1}{4}p - \frac{1}{4}qp - \frac{1}{16}p^3 - \frac{1}{2}r + x$, eſt æquale quadrato ex AI, ſcilicet quadrato ex $x^2 - \frac{1}{2}px - \frac{1}{2}q - \frac{1}{8}p^2 - \frac{1}{2}$, erit igitur æquatio

$- x^2$

$$\begin{array}{llllll}
-x^2 & +\frac{1}{2}px & +e^2 & \infty\, x^4 - px^3 & -qx^2 & +pqx & +\frac{1}{4}q^2, \\
 & +\frac{1}{2}qp & -\frac{1}{16}p^2 & & +\frac{1}{4}p^2 & +\frac{1}{8}p^3 & +\frac{1}{8}p^2q \\
 & +\frac{1}{8}p^3 & -\frac{1}{8}qp^2 & & -\frac{1}{4}p^2 & +\frac{1}{2}p & +\frac{1}{64}p^4 \\
 & +r & -\frac{1}{32}p^4 & & -1 & & +\frac{1}{2}q \\
 & & -\frac{1}{16}q^2p^2 & & & & +\frac{1}{8}p^2 \\
 & & -\frac{1}{32}qp^4 & & & & +\frac{1}{4} \\
 & & -\frac{1}{4}pr & & & & \\
 & & -\frac{1}{4}qpr & & & & \\
 & & -\frac{1}{256}p^6 & & & & \\
 & & -\frac{1}{16}p^3r & & & & \\
 & & -\frac{1}{4}r^2 & & & &
\end{array}$$

ſcilicet

$$\begin{array}{llllll}
-x^2 & +\frac{1}{2}px & +\frac{1}{4}q^2 & \infty\, x4 - px^3 & -qx^2 & +pqx & +\frac{1}{4}q^2; \\
 & +\frac{1}{2}qp & +\frac{1}{8}p^2q & & +\frac{1}{4}p^2 & +\frac{1}{8}p^3 & +\frac{1}{8}p^2q \\
 & +\frac{1}{8}p^3 & +\frac{1}{64}p^4 & & -\frac{1}{4}p^2 & +\frac{1}{2}p & +\frac{1}{64}p^4 \\
 & +r & +\frac{1}{2}q & & -1 & & +\frac{1}{2}q \\
 & & +\frac{1}{8}p^2 & & & & +\frac{1}{8}p^2 \\
 & & +\frac{1}{4} & & & & +\frac{1}{4} \\
 & & +\text{ſ} & & & &
\end{array}$$

Et ablatis ex ea quantitatibus communibus, & iis, quæ ſe mutuò tollunt, manifeſtabitur æquatio propoſita $rx + \text{ſ} \infty x^4 - px^3 - qx^2$, hoc eſt $x^4 - px^3 - qx^2 - rx - \text{ſ} \infty 0$. Q. E. D.

Non minori quidem facilitate è duabus æquationibus primo loco repertis, idem aſſequemur.

Inventa eodem, quo ſuprà modo, prima ex ipſis $x^2 - \frac{1}{2}px \infty ay$; Quia CE fuit poſita ∞y,

ſi ab ea auferatur CQ $\propto \frac{1}{2}q + \frac{1}{8}p^2 + \frac{1}{2}$; reliqua portio erit QE, ſive AI ad diametrum circuli ordinata $\propto y - \frac{1}{2}q - \frac{1}{8}p^2 - \frac{1}{2}$. Poſitis deinde cæteris, ut ſuprà, invenietur æquatio.

$$\begin{array}{llllll}
-x^2 & +\frac{1}{2}px & +e^2 & \propto y^2 - & qy & +\frac{1}{4}q^2\,, \\
 & +\frac{1}{2}qp & -\frac{1}{16}p^2 & & -\frac{1}{4}p^2 & +\frac{1}{8}p^2q \\
 & +\frac{1}{8}p^3 & -\frac{1}{8}qp^2 & & -a & +\frac{1}{64}p^4 \\
 & +r & -\frac{1}{32}p^4 & & & +\frac{1}{2}q \\
 & & -\frac{1}{16}q^2p^2 & & & +\frac{1}{8}p^2 \\
 & & -\frac{1}{32}qp^4 & & & +\frac{1}{4} \\
 & & -\frac{1}{4}pr & & & \\
 & & -\frac{1}{4}qpr & & & \\
 & & -\frac{1}{256}p^6 & & & \\
 & & -\frac{1}{16}p^3r & & & \\
 & & -\frac{1}{4}r^2 & & &
\end{array}$$

ideſt

$$\begin{array}{llllll}
-x^2 & +\frac{1}{2}px & +\frac{1}{4}q^2 & \propto y^2 - & qy & +\frac{1}{4}q^2\,,\text{\& dēptis} \\
 & +\frac{1}{2}qp & +\frac{1}{8}p^2q & & -\frac{1}{4}p^2 & +\frac{1}{8}p^2q \\
 & +\frac{1}{8}p^3 & +\frac{1}{64}p^4 & & -a & +\frac{1}{64}p^4 \\
 & +r & +\frac{1}{2}q & & & +\frac{1}{2}q \\
 & & +\frac{1}{8}p^2 & & & +\frac{1}{8}p^2 \\
 & & +\frac{1}{4} & & & +\frac{1}{4} \\
 & & +\text{ſ} & & &
\end{array}$$

communibus, erit

$$\begin{array}{lllll}
-x^2 & +\frac{1}{2}px & +\text{ſ} \propto y^2 & - & qy\,; \\
 & +\frac{1}{2}qp & & & -\frac{1}{4}p^2 \\
 & +\frac{1}{8}p^3 & & & -a \\
 & +r & & &
\end{array}$$

addi-

addita poſtea utrique parti hujus æquationis prima inventa $x^2 - \frac{1}{2} px \propto ay$,

habebitur

$$\begin{array}{l} \frac{1}{2} qpx \\ + \frac{1}{8} p^3 \\ + r \end{array} + \text{ſ} \propto y^2 - \begin{array}{l} qy, \\ - \frac{1}{4} p^2 \\ \end{array}$$

ſive

$$\begin{array}{l} \frac{1}{2} \frac{qp}{a} x \\ + \frac{1}{8} \frac{p^3}{a^2} \\ + r \end{array} + a\text{ſ} \propto y^2 - \begin{array}{l} qy; \\ - \frac{1}{4} \frac{p^2}{a} \\ \end{array}$$

ideſt

$$\begin{array}{l} \frac{1}{2} \frac{qp}{a} x \\ + \frac{1}{8} \frac{p^3}{a^2} \\ + r \end{array} + a\text{ſ} + \begin{array}{l} qy \\ + \frac{1}{4} \frac{p^2}{a} \\ \end{array} \propto y^2;$$

& demum reſtituto in locum $\frac{1}{2} px + ay$ ejus valore x^2, apparebit altera æquatio

$$\begin{array}{l} \frac{q}{a} x^2 \\ + \frac{1}{4} \frac{p^2}{a^2} \end{array} + rx + a\text{ſ} \propto y^2.$$ Q. E. iterum D.

Eadem ratione oſtendi poteſt VZ, XY, & NO falſas eſſe ejuſdem æquationis radices, ſumptis ſingulis ipſarum $\propto -x$, & cæteris, ut ſuprà, poſitis.

Hæc methodus eſt univerſalis ad inveniendam conſtructionem æquationum omnium trium, & quatuor dimenſionum, & locum non tantùm ſibi vindicat in iis, in quibus omnes extant termini,

sed

ſed etiam in illis, quæ aliquibus eorum carent, imò ſi defuerit ſecundus, facilior evadit calculus; non enim opus erit efformare quadratum ex latere $x^2 - \frac{1}{2} px$, vel prout neceſſitas urget, ex $x^2 + \frac{1}{2} px$, ut in æquationibus, in quibus ſecundus reperitur terminus, ſed tantùm ſufficiet effingere plano-planum a^2y^2 æquale utrique parti æquationis, & cætera, ut ſuprà, dummodo æquatio trium dimenſionum ad quatuor uſque extollatur.

Sit invenienda conſtructio æquationis

$$x^4 \quad * - qx^2 + rx - ſ,$$

ſive $x^4 \infty * + qx^2 - rx + ſ$ explicabilis juxtà ſui naturam de quatuor radicibus, tribus nimirum veris, & quarta falſa, veris æquali, vel de duabus tantùm inæqualibus, altera ſcilicet vera, altera autem falſa, cum reliquæ duæ imaginariæ exiſtant.

Ponatur plano-planum a^2y^2 æquale utrique parti æquationis, poſita, ut ſuprà, a pro unitate, & habebitur $x^4 \infty a^2y^2$, ſive $x^2 \infty ay$ ad parabolam, & $qx^2 - rx + ſ \infty a^2y^2$, ſive $aqx^2 - a^2rx + a^3ſ \infty a^2y^2$, hoc eſt $\frac{qx^2}{a} - rx + aſ \infty y^2$ ad hyperbolam, vel ad ellipſim, ſi terminus qx^2 ſigno $-$ afficiatur, ſive ad circulum, ſi a non unitati, ſed $\sqrt{q}$ ponatur æqualis, ut videre eſt in æquatione $x^4 * + qx^2 - rx + ſ$, hoc eſt $x^4 \infty * - qx^2 + rx - ſ$. Sed ſi fingatur quadratum ex latere $x^2 - \frac{1}{2} q$, ſcilicet $x^4 - qx^2 + \frac{1}{4} q^2$, & portio ipſius $- qx^2 + \frac{1}{4} q^2$ addatur utrique parti æquationis, ut evaneſcat terminus qx^2, invenientur æquationes $x^2 - \frac{1}{2} q \infty ay$ ad parabolam,

lam, & $-rx \begin{array}{l} + a\text{ſ} \\ + \frac{1}{4}q^2 \end{array} \infty y^2$ ſimiliter ad parabolam, ex qua facilis eſt ad circulum tranſitus, ut circuli, & parabolæ interſectione æquatio conſtruatur.

At ad methodum redeuntes, pro hyperbola circulum habebimus, ſi in locum x^2 ſubrogabimus ejus valorem ay; nam ex illa æquatione emerget hæc altera $qy - rx + a\text{ſ} \infty y^2$, hoc eſt $-rx + a\text{ſ} \infty y^2 - qy$, ſive $-rx + \text{ſ} \infty y^2 - qy$, ex qua ſi priorem deducamus $x^2 \infty ay$, ad circulum æquatio orietur $-x^2 - rx + \text{ſ} \infty y^2 \begin{array}{l} - qy \\ - a \end{array}$

Addito enim utrique parti quadrato dimidii quantitatis notæ ſecundi termini, ubi habetur y^2, ſcilicet quadrato ex $-\frac{1}{2}q - \frac{1}{2}$, vel ex $\frac{1}{2}q + \frac{1}{2}$, nimirum $\frac{1}{4}q^2 + \frac{1}{2}q + \frac{1}{4}$, erit

$$-x^2 - rx \begin{array}{l} + \frac{1}{4}q^2 \\ + \frac{1}{2}q \\ + \frac{1}{4} \\ + \text{ſ} \end{array} \infty y^2 \begin{array}{l} - qy \\ - a \\ \\ \\ \end{array} \begin{array}{l} + \frac{1}{4}q^2 \\ + \frac{1}{2}q \\ + \frac{1}{4} \\ \\ \end{array}$$

Rurſus addito illi æquationis parti, in qua reperitur $-x^2$, quadrato alterius dimidii notæ quantitatis ſecundi termini, ſcilicet quadrato ex $\frac{1}{2}r$, hoc eſt $\frac{1}{4}r^2$, & ablato deinde ex aggregato $\frac{1}{4}q^2 + \frac{1}{2}q + \frac{1}{4} + \text{ſ} + \frac{1}{4}r^2$, quod ponitur ∞e^2, ut mutetur æquatio in $-x^2 - rx + e^2 - \frac{1}{4}r^2$, circulus ſtatim manifeſtabitur, ejuſque ſemidiameter erit $\infty \sqrt{e^2}$, ſcilicet $\infty \sqrt{\frac{1}{4}q^2 + \frac{1}{2}q + \frac{1}{4} + \text{ſ} + \frac{1}{4}r^2}$. major diametri pars, cui axis para-

parabolæ perpendiculariter inſiſtit $\infty \sqrt{e^2} + \frac{1}{2}r$. minor $\infty \sqrt{e^2} - \frac{1}{2}r$, & ordinata ad diametrum $\infty \sqrt{\frac{1}{4}q^2 + \frac{1}{2}q + \frac{1}{4} + ſ}$, ſcilicet æqualis lateri quadrati ex $-\frac{1}{2}q = \frac{1}{2}$, ſive ex $\frac{1}{2}q + \frac{1}{2}$, unà cum quadrato ex $\sqrt{ſ}$, quæ reperictur $\infty \frac{1}{2}q + \frac{1}{2}$, ſi deficiat ultimus terminus, hoc eſt æquatio ſit trium dimenſionum.

Et hæc eſt propoſitæ æquationis reſolutio.

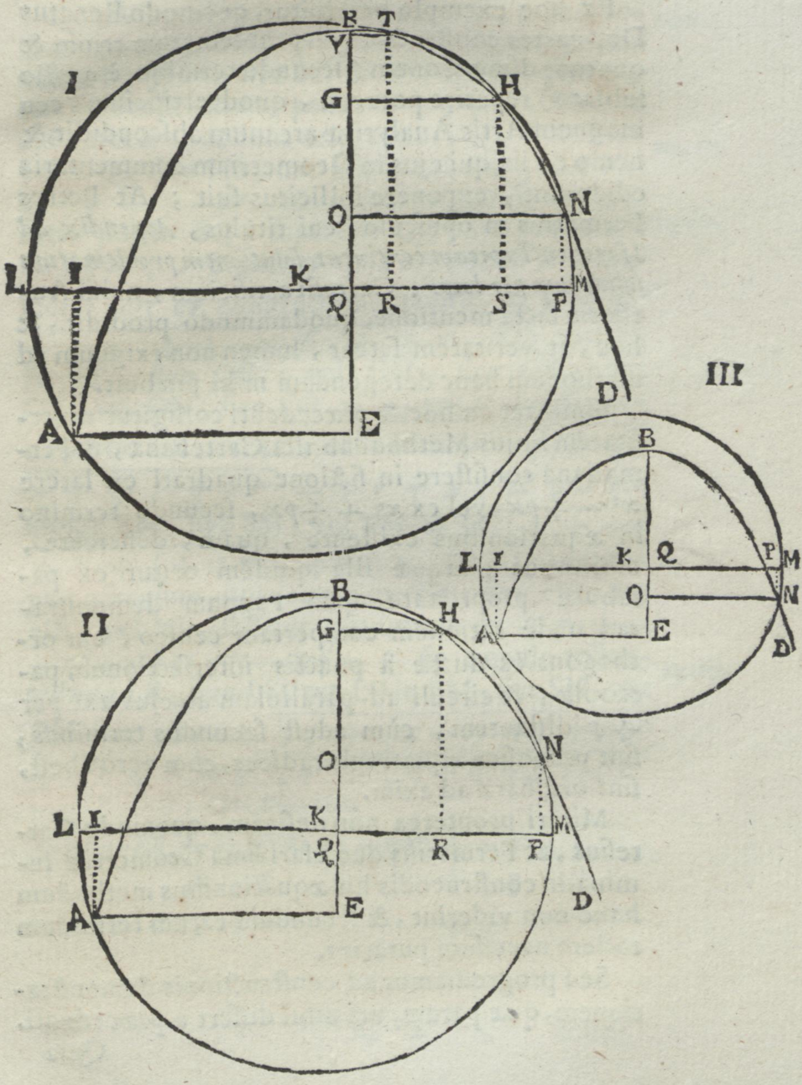
Deſcribatur pro ejus conſtructione parabola ABD, cujus parameter a, ſive unitas, ſit BE axis ∞y. Sumatur portio BQ $\infty \frac{1}{2}q + \frac{1}{2}$, ducatur ex puncto Q ad rectos angulos QK $\infty \frac{1}{2}r$, & producta in rectum, conſtituatur KL ſemidiameter circuli, ex centro K deſcripti $\infty \sqrt{e^2}$, nimirum æqualis lateri quadrati, compoſiti ex quadratis BQ, QK, & $\sqrt{ſ}$, ſimul ſumptis, hoc eſt $\infty \sqrt{\frac{1}{4}q^2 + \frac{1}{2}q + \frac{1}{4} + ſ + \frac{1}{4}r^2}$. Dico parabolam ab hoc circulo in quatuor punctis A, T, H, N ſecari, vel in tribus A, H, N, quod accidit, cùm duæ ex veris radicibus inter ſe æquantur (tunc enim punctum H congruit cum puncto T, & circulus ibi parabolam tangit), vel demum in duobus A, & N, ſi duæ ex ipſis imaginariæ exiſtant.

Fig. I. Fig. II.

Fig. III.

Si igitur ex his punctis ad parabolæ axim ordinatæ ducantur, erunt hæ propoſitæ æquationis radices, AE nimirum falſa, cæteræ autem veræ.

Ex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Ex hoc exemplo percipitur, quomodo Renatus Des-cartes conſtructionem problematum trium,& quatuor dimenſionum (ſecundo termino è medio ſublato) reperire potuerit , quod artificium , ceu magnum Artis Analyticæ arcanum abſcondidit,& nemo ex iis,qui ejus in Geometriam commentaria ediderunt , exponere ſollicitus fuit ; At Petrus Fermatius in opuſculo , cui titulus , *Appendix ad Iſagogen Topicam continens ſolutionem problematum ſolidorum per locos* , Carteſii artificium , nullâ Authoris factâ mentione, quodammodo prodidit , & hoc , ut veritatem fatear , lumen non exiguum ad methodum hanc detegendam mihi præbuit.

Similiter ex hoc,& præcedenti colligitur diverſitatem hujus Methodi ab illa Carteſiana , & Fermatiana conſiſtere in fictione quadrati ex latere $x^2 - \frac{1}{2}px$, vel ex $x^2 + \frac{1}{2}px$, ſecundo termino in æquationibus exiſtente , quam , deficiente, omittimus ; atquæ illa quidem oritur ex parabolæ proprietate apud Pappum demonſtrata ; undè rationem compertam cenſeo , cur orthogonales ductæ à punctis interſectionum parabolæ , & circuli ad parallelam ab ejus axe per $\frac{1}{4}p$ diſtantem , cùm adeſt ſecundus terminus , ſint propoſitæ æquationis radices, cùm verò abeſt, ſint ordinatæ ad axim.

Mirari propterea non deſinam , quomodo Carteſius , ac Fermatius duo clariſſima Geometriæ lumina,in conſtruendis his æquationibus methodum hanc non viderint , & ſecundum earum terminum tollere neceſſum putarint.

Sed progrediamur ad conſtructionis demonſtrationem, quæ parum, vel nihil differt à præcedenti.

Quia

Quia VT, ſive GH, vel ON eſt $\propto x$, & parabolæ parameter ponitur $\propto a$, ſive unitati, erit portio axis BV, vel BG, ſive BO $\propto x^2$. Rurſus quia BQ fuit ex conſtructione poſita $\propto \frac{1}{2}q + \frac{1}{2}$, ſi ab ea auferatur x^2, reliqua erit portio VQ, ſive TR, vel GQ, ſive HS, aut HR, vel OQ, ſive NP $\propto \frac{1}{2}q + \frac{1}{2} - x^2$. Sin autem ex BO $\propto x^2$ auferatur portio BQ, ſupererit QO, ſive PN $\propto x^2 - \frac{1}{2}q - \frac{1}{2}$. Similiter quia LK circuli ſemidiameter fuit ex conſtructione poſitus $\propto \sqrt{\frac{1}{4}q^2 + \frac{1}{2}q + \frac{1}{4} + \text{ſ} + \frac{1}{4}r^2}$. KQ $\propto \frac{1}{2}r$, & VT, ſive QR, vel GH, ſive QS, aut QR, vel ON, ſive QP $\propto x$, erit major diametri portio LR, ſive LS, aut LP $\propto \sqrt{\frac{1}{4}q^2 + \frac{1}{2}q + \frac{1}{4} + \text{ſ} + \frac{1}{4}r^2} + \frac{1}{2}r + x$, ac proinde minor RM, ſive SM, vel PM $\propto \sqrt{\frac{1}{4}q^2 + \frac{1}{2}q + \frac{1}{4} + \text{ſ} + \frac{1}{4}r^2} - \frac{1}{2}r - x$. Cùm igitur propter circulum, quadratum ex TR, ſive HS, aut HR, vel NP, ſcilicet quadratum ex $\frac{1}{2}q + \frac{1}{2} - x^2$, vel ex $x^2 - \frac{1}{2}q - \frac{1}{2}$ ſit æquale rectangulo ex LR, ſive LS, aut LP in RM, ſive SM, aut PM, hoc eſt ex $\sqrt{\frac{1}{4}q^2 + \frac{1}{2}q + \frac{1}{4} + \text{ſ} + \frac{1}{4}r^2} + \frac{1}{2}r + x$ in $\sqrt{\frac{1}{4}q^2 + \frac{1}{2}q + \frac{1}{4} + \text{ſ} + \frac{1}{4}r^2} - \frac{1}{2}r - x$, erit æquatio

Fig. I.
Fig. II.
Fig. III.
Fig. II.
Fig. III.

$$\begin{array}{lllll} -x^2 - rx & + \frac{1}{4}q^2 & \propto x^4 & - qx^2 & + \frac{1}{4}q^2, \\ & + \frac{1}{2}q & & - 1 & + \frac{1}{2}q \\ & + \frac{1}{4} & & & + \frac{1}{4} \\ & + \text{ſ} & & & \end{array}$$

& deletis quantitatibus communibus, habebitur $-rx+ſ \infty x^4-qx^2$, ſcilicet $x^4 \infty * +qx^2 -rx+ſ$. Q.E.D.

Atque hæc demonſtratio ſimplicior Carteſiana mihi videtur, nàm illa conſiſtit in comparandis intrà circulum hypotenuſis duorum orthogonalium triangulorum, veluti ſemidiametris; hæc verò oritur ex proprietate earumdem curvarum conſtructionem efficientium.

Eadem ratione conſtabit AE falſam eſſe propoſitæ æquationis radicem, at tribus veris æqualem, ſatis luculentèr demonſtravit Franciſcus à Schooten Analyſcos beneficio in commentariis Carteſianæ Geometriæ.

Lib. 3.

Et hæc eſt methodus, quam pro conſtruendis æquationibus omnibus explanandam ſuſcepimus, quæ non ſolum ſimplicior eſſe deprehenditur eâ, qua uſus fuit Sluſius in ſua Analyſi pro conſtructione æquationum, ſed etiam illâ, quam inveniſſe Backerum in Regula ab ipſo Centrali appellata Sturmius refert, & hoc, ut prætercam, tam Sluſium, quam Backerum Renati veſtigiis inſiſtentes, eodem defectu unà cum illo laboraſſe, poſitam æquationem de duabus tantùm radicibus explicantes, cùm de quatuor etiam juxtà ſui naturam explicabilis eſſe potuiſſet.

Cæterùm, ne quid ad methodi amplitudinem deſit, proponatur inveſtiganda conſtructio æquationis $x^4 * +qx^2 * -ſ$, ſive $x^4 \infty * -qx^2 * +ſ$, de duabus æqualibus explicabilis radicibus, altera nimirum falſa, altera vera, reliquis duabus imaginariis exiſtentibus; quæ, & ſi beneficio Geometriæ, quam planam vocant, nullo negotio haberi

poſſint, tamen ob originem æquationis ſolidam, expoſita methodo non minori facilitate invenientur.

Æquetur plano-planum a^2y^2 utrique parti æquationis, & habebitur $x^4 \infty a^2y^2$, ſcilicet $x^2 \infty ay$ ad parabolam ex una parte, ex altera verò $-\frac{qx^2}{a}$ $+ a\text{ſ} \infty y^2$ ad circulum, ſi a, ſive unitas ſit ∞q, ſin autem major, vel minor, ad ellipſim.

Quòd ſi formetur quadratū ex latere $x^2 + \frac{1}{2}q$, ut deſtruatur terminus æquationis $-qx^2$, invenientur $x^2 + \frac{1}{2}q \infty ay$ ad parabolam, & $\frac{1}{4}q^2$ $+ a\text{ſ} \infty y^2$ ad hyperbolam, ex qua ſi auferatur, æquatio ad parabolam, alia ad circulum emerget.

$$\begin{array}{lll} -x^2 & +\frac{1}{4}q^2 & \infty\, y^2 - ay. \\ & -\frac{1}{2}q & \\ & +\; a\text{ſ} & \end{array}$$

Sed ut ellipſis in circulum evadat, eadem ratione, qua in præcedentibus exemplis, procedemus.

Primò in locum x^2 ſubrogabimus ejus valorem ay, & habebimus æquationem $-qy + a\text{ſ} \infty y^2$. Deinde auferemus ex ea $x^2 \infty ay$, & ad circulum æquatio orietur $-x^2 - qy + a\text{ſ} \infty y^2 - ay$, hoc eſt

$$\begin{array}{llll} -x^2 + a\text{ſ} \infty y^2 & +\,qy & \text{, ſive } -x^2 + a\text{ſ} \infty y^2 + by, \\ & -\,a & \end{array}$$

poſita $b \infty \overline{q-a}$, ſi q major ſit quàm a.

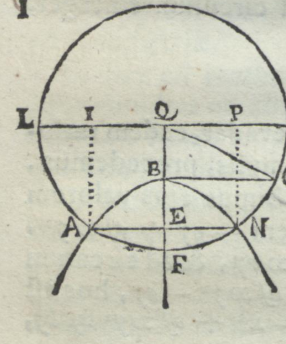
Addito enim utrique ejus parti quadrato ex dimidio $\overline{q-a}$, ideſt $\overline{q-1}$, ſive b, ſcilicet $\frac{1}{4}q^2$ $-\frac{1}{2}q + \frac{1}{4}$, vel $\frac{1}{4}b^2$, erit

$$\begin{array}{lllll} -x^2 & +\frac{1}{4}q^2 & \infty\, y^2 & +\,qy & +\frac{1}{4}q^2 \;, \\ & -\frac{1}{2}q & & -\,a & -\frac{1}{2}q \\ & +\frac{1}{4} & & & +\frac{1}{4} \\ & +\;\text{ſ} & & & \end{array}$$

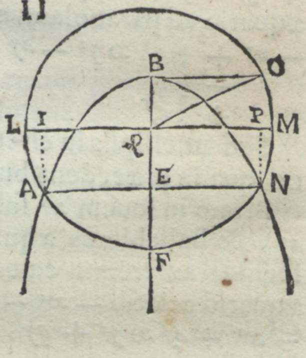
hoc eſt $-x^2 \begin{array}{l} +\frac{1}{4}b^2 \\ +\; ſ \end{array} \propto y^2 + by + \frac{1}{4}b^2$, atque hinc circulus emergit, cujus centrum erit punctum Q, per quod tranſit axis parabolæ ultrà verticem productus, ſemidiameter $\propto \sqrt{\frac{1}{4}q^2 - \frac{1}{2}q + \frac{1}{4} + ſ}$, ſive $\propto \sqrt{\frac{1}{4}b^2 + ſ}$, & ordinata ad diametrum latus ex quadrato $\frac{1}{2}q - \frac{1}{2}$, ſive $\frac{1}{2}b$, unà cum quadrato ex $\sqrt{ſ}$, hoc eſt idem ſemidiameter. *Fig. I.*

Et hæc eſt reſolutio propoſitæ æquationis, ex qua conſtructio facillimè deducitur.

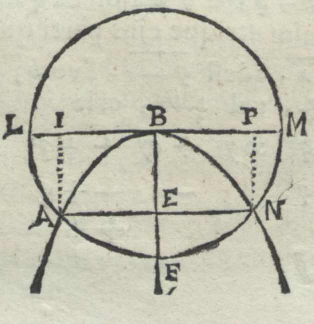
I



II



III



Deſcri-

Deſcribatur parabola ABN, cujus parameter a, ſive unitas, & axis BF $\infty\, y$, ſumatur portio BQ ultrà verticis punctum $\infty\, \frac{1}{2}b$, hoc eſt $\infty\, \frac{1}{2}q - \frac{1}{2}$. Ducatur ex puncto Q orthogonalis ad eam QL $\infty\, \sqrt{\frac{1}{4}b^2 + \text{ſ}}$, nimirũ $\infty\, \sqrt{\frac{1}{4}q^2 - \frac{1}{2}q + \frac{1}{4} + \text{ſ}}$, ideſt æqualis lateri ex ſumma quadratorum BQ, & $\sqrt{\text{ſ}}$, ſive BO. Conſtituatur hæc recta ſemidiameter circuli, cujus centrum Q. Dico hunc eſſe circulum quæſitum, qui parabolam citrà verticem in duobus punctis A, & N ſecabit, ex quibus ſi ordinatæ ad axim AE, NE ducantur, erunt propoſitæ æquationis radices, altera ſcilicet vera, altera falſa.

Quòd ſi q ſit minor quàm a, debebit ſumi $-b \,\infty\, q - a$, & addito quadrato ex ipſius dimidio utrique parti æquationis, orietur æquatio ad circulum $-x^2 \begin{array}{l}+\frac{1}{4}b^2\\+\text{ſ}\end{array} \infty\, y^2 - by + \frac{1}{4}b^2$, cujus centrum erit in puncto axis citrà verticem diſtans per $-\frac{1}{2}b$, hoc eſt per $-\frac{1}{2}q + a$. Fig.II.

Si verò ſit æqualis a, habebitur, ut diximus, ad circulum æquatio $-x^2 + a\text{ſ} \,\infty\, y^2$, & ejus centrum erit in ipſo parabolæ vertice, quod ex conſtructionibus clarius manifeſtum fit. Sed ad jam poſitæ conſtructionis demonſtrationem progrediamur. Fig.III.

Quia AE eſt $\infty\, x$, & parabolæ parameter $\infty\, a$, ſive unitati, erit portio axis EB $\infty\, x^2$. Rurſus, quia altera portio BQ ultrà verticem producta poſita eſt $\infty\, \frac{1}{2}b$, hoc eſt $\infty\, \frac{1}{2}q - \frac{1}{2}$, erit tota EQ, ſive AI $\infty\, x^2 + \frac{1}{2}b$, ſcilicet $\infty\, x^2 + \frac{1}{2}q - \frac{1}{2}$. Ad hæc, quia circuli ſemidiameter LQ, ſive QM poſitus fuit ex conſtructione $\infty\, \sqrt{\frac{1}{4}b^2 + \text{ſ}}$, nimirum Fig.I.

rum $\infty \sqrt{\frac{1}{4} q^2 - \frac{1}{2} q + \frac{1}{4} + \text{ſ}}$, & AE, ſive IQ $\infty\, x$, erit MI major diametri portio $\infty \sqrt{\frac{1}{4} b^2 + \text{ſ}} + x$, ideſt $\infty \sqrt{\frac{1}{4} q^2 - \frac{1}{2} q + \frac{1}{4} + \text{ſ}} + x$, & LI minor $\infty \sqrt{\frac{1}{4} b^2 + \text{ſ}} - x$, ſcilicet $\infty \sqrt{\frac{1}{4} q^2 - \frac{1}{2} q + \frac{1}{4} + \text{ſ}} - x$. Propterea cùm propter circulum, quadratum ex EQ, ſive AI ſit æquale rectangulo ex MI in IL, emerget æquatio

$$\begin{array}{llll} -x^2 & + \frac{1}{4} b^2 \infty x^4 + bx^2 & + \frac{1}{4} b^2, \\ & + \text{ſ} & \end{array}$$

ſcilicet

$$\begin{array}{llllll} -x^2 & + \frac{1}{4} q^2 & \infty & x^4 & + qx^2 & + \frac{1}{4} q^2, \\ & - \frac{1}{2} q & & & - 1 & - \frac{1}{2} q \\ & + \frac{1}{4} & & & & + \frac{1}{4} \\ & + \text{ſ} & & & & \end{array}$$

& deletis quantitatibus cōmunibus, $\text{ſ} \infty x^4 + qx^2$; nimirum $x^4 \infty * - qx^2 * + \text{ſ}$. Q. E. D.

Eadem ratione demonſtrabitur NE falſam eſſe propoſitæ æquationis radicem, & quod ea ſit veræ æqualis, ex ipſa conſtructione patet. Atque hæc de æquationibus quatuor graduum dicta ſufficiant. Deſcendamus nunc ad conſtructionem æquationum trium dimenſionum.

Proponatur invenienda conſtructio æquationis $x^3 \infty \text{ſ}$, de unica duntaxat vera radice explicabilis, quæ oritur ex problemate de duabus medio loco proportionalibus inter duas rectas datas inveniendis.

Sumatur a pro unitate, & æquatio ad quartum gradum exaltetur.

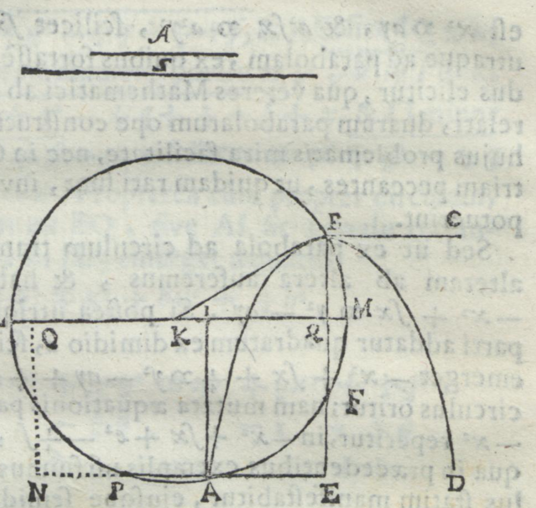
Ponatur deinde utraque ipſius pars $\infty\, a^2 y^2$, & duæ invenientur æquationes $x^4 \infty a^2 y^2$, hoc

eſt

eſt $x^2 \infty ay$, & $a^2ſx \infty a^2y^2$, ſcilicet $ſx \infty y^2$: utraque ad parabolam, ex quibus fortaſſè methodus elicitur, qua veteres Mathematici ab Eutocio relati, duarum parabolarum ope conſtructionem hujus problematis mira facilitate, nec in Geometriam peccantes, ut quidam rati ſunt, inveſtigare potuerint.

Sed ut ex parabola ad circulum tranſeamus, alteram ab altera auferemus, & habebimus $-x^2 + ſx \infty y^2 - ay$. Si poſtea utrique ejus parti addatur quadratum ex dimidio a, ſcilicet $\frac{1}{4}$, emerget $-x^2 + ſx + \frac{1}{4} \infty y^2 - ay + \frac{1}{4}$, ex qua circulus oritur; nam mutata æquationis parte, ubi $-x^2$ reperitur, in $-x^2 + ſx + e^2 - \frac{1}{4}ſ^2$, regula, qua in præcedentibus exemplis uſi fuimus, circulus ſtatim manifeſtabitur, ejuſque ſemidiameter erit $\infty \sqrt{e^2}$, hoc eſt $\infty \sqrt{\frac{1}{4} + \frac{1}{4}ſ^2}$, major diametri pars, cui axis parabolæ ad rectos angulos inſiſtit $\infty \sqrt{e^2} + \frac{1}{2}ſ$, minor $\infty \sqrt{e^2} - \frac{1}{2}ſ$, & ordinata ad diametrum $\infty \frac{1}{2}$.

Et hæc eſt reſolutio propoſitæ æquationis.



Deſcribatur pro ejus conſtructione parabola ABD, cujus parameter *a*, ſive unitas. Sumatur portio axis BQ $\infty \frac{1}{2}$. Ducatur ex puncto Q orthogonalis QK $\infty \frac{1}{2} ſ$, & producta ſumatur KL æqualis lateri ex quadratis KQ, & QB ſimul ſumptis, nimirum $\infty \sqrt{e^2}$, hoc eſt $\infty \sqrt{\frac{1}{4} + \frac{1}{4} ſ^2}$. Conſtituatur hæc recta circuli ſemidiameter, facto centro in K; dico hunc circulum eſſe quæſitum, qui parabolam, comprehenſo verticis puncto B, ſecabit in A, ex quo ſi ducatur AE ordinata ad axim, erit hæc propoſitæ æquationis radix, ac proinde prima ex duabus medio loco proportionalibus inter rectas datas, quarum altera erit *a*, altera verò *ſ*.

De-

Demonſtratio.

Quia LK poſita eſt $\infty \sqrt{\frac{1}{4} + \frac{1}{4}\text{ſ}^2}$. KQ $\infty \frac{1}{2}\text{ſ}$, & AE, ſive IQ inventa $\infty\, x$, erit LI $\infty \sqrt{\frac{1}{4} + \frac{1}{2}\text{ſ}^2} + \frac{1}{2}\text{ſ} - x$, & IM $\infty \sqrt{\frac{1}{4} + \frac{1}{4}\text{ſ}^2} - \frac{1}{2}\text{ſ} + x$. Quoniam ſimiliter (poſita a pro unitate) portio axis BE propter parabolam, eſt $\infty\, x^2$, dempta ex ea BQ, reliqua portio QE, ſive IA erit $\infty\, x^2 - \frac{1}{2}$. Cùm igitur ob circulum, quadratum ex IA ſit æquale rectangulo ex LI in IM, emerget æquatio $x^4 - x^2 + \frac{1}{4} \infty - x^2 + \text{ſ}x + \frac{1}{4}$, & deletis ſuperfluis, ſupererit $x^4 \infty\, \text{ſ}x$, hoc eſt $x^3 \infty\, \text{ſ}$. Q. E. D.

Hæc igitur demonſtratio nedum locum habet, cùm a pro unitate, ſed cùm pro altera ex datis accipitur; nam ſive ea ſit unitas, ſive altera ex datis, ſemper erit parabolæ parameter, & portio axis BQ æqualis ipſius dimidio; reliqua verò ſ æqualis OQ, ſive NE, hoc eſt duplo portionis KQ ſemidiametri circuli.

Altera Demonſtratio Veterum more.

Quia in poſita figura rectangulum PEA æquatur rectangulo BEF, addito communi quadrato ex AE, ſive NP, erit rectangulum NEA æquale rectangulo BEF unà cum quadrato ex AE. Quia ſimiliter hoc quadratum æquatur rectangulo EBC, ſive EBF, erit NEA rectangulum æquale rectangulis BEF, & EBF, hoc eſt æquale quadrato ex EB; propterea igitur, ut AE ad EB, ita EB ad NE, ſive QO, vel s; undè cùm ſit per parabolam, ut BC, ſive $\mathcal{A}$ ad AE, ita AE ad EB, erunt quatuor rectæ $\mathcal{A}$. AE. EB, & s continuò proportionales, ac proinde AE prima ex mediis quæſitis. Q. E. D.

Rurſus proponatur conſtruenda æquatio $x^3 \infty * -rx + ſ$, de unica tantùm vera radice explicabilis, cùm reliquæ duæ imaginariæ exiſtant.

Exaltata hac ad quartum gradum, & poſita utrâque ejus parte ∞a^2y^2, habebitur $x^2 \infty ay$ ad parabolam, & $-rx^2 + ſx \infty a^2y^2$, ſcilicet $-arx^2 + a^2ſx \infty a^2y^2$, nimirum $-\frac{rx^2}{a} + ſx \infty y^2$ ad circulum, ſi a, ſive unitas ſit ∞r; ad ellipſim verò, ſi major, aut minor.

Sed ſi tàm in hoc exemplo, quàm in ſequentibus ad cubicas æquationes ſpectantibus, formetur quadratum ex latere $x^2 + \frac{1}{2}r$, aut $x^2 - \frac{1}{2}r$, ut evaneſcat terminus $-rx^2$, vel $+rx^2$, æquationes ad duas parabolas invenientur, nimirũ $x^2 + \frac{1}{2}r \infty ay$, & $-ſx + \frac{1}{4}r^2 \infty y^2$, ſive $x^2 - \frac{1}{2}r \infty ay$, & $-ſx + \frac{1}{4}r^2 \infty y^2$, vel $x^2 - \frac{1}{2}r \infty ay$, & $ſx + \frac{1}{4}r^2 \infty y^2$, aut $x^2 + \frac{1}{2}r \infty ay$, & $ſx + \frac{1}{4}r^2 \infty y^2$; ex quibus locis facilis problematum omnium ſolidorum conſtructio inveniri poteſt, & fœliciter promoveri methodus, qua veteres Mathematici problema de duabus mediis proportionalibus, duarum parabolarum ope, conſtruxerunt.

Quòd ſi poſtea circuli, & parabolæ interſectione propoſitas æquationes conſtruere quis velit, facilis erit ex altera harum parabolarum ad circulum tranſitus, regula ſuprà indicata.

At ad methodum redeuntes, ut ex ellipſi ad circulum tranſeamus; Primò in locum x^2 ſubrogabimus ejus valorem ay, & habebimus æquationem $-ry + ſx \infty y^2$, nimirum $ſx \infty y^2 + ry$. Deinde ex ea auferemus $x^2 \infty ay$, & altera exorietur æquatio $-x^2 + ſx \infty y^2 \begin{smallmatrix}+r\\-a\end{smallmatrix} y$,

ſive

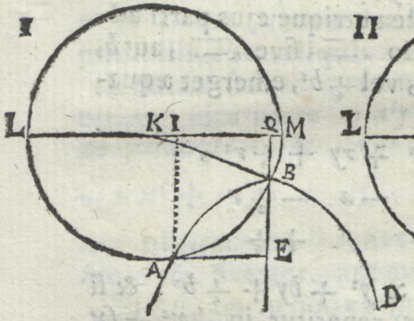
ſive $- x^2 + ſx \infty y^2 + by$, poſita $b \infty \overline{r-a}$, ſi r major ſit quàm a; ſi poſtea utrique ejus parti addatur quadratū ex dimidio $\overline{r-a}$, ſive $\overline{r-1}$, aut b, ſcilicet $\frac{1}{4}r^2 - \frac{1}{2}r + \frac{1}{4}$, vel $\frac{1}{4}b^2$, emerget æquatio ad circulum

$$\begin{array}{lll} -x^2 + ſx & + \frac{1}{4}r^2 \infty y^2 & + ry + \frac{1}{4}r^2, \\ & - \frac{1}{2}r & - a \quad - \frac{1}{2}r \\ & + \frac{1}{4} & \qquad\;\; + \frac{1}{4} \end{array}$$

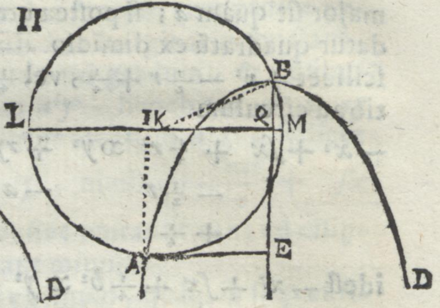
ideſt $- x^2 + ſx + \frac{1}{4}b^2 \infty y^2 + by + \frac{1}{4}b^2$, & ſi mutetur pars, in qua $- x^2$ reperitur, in $- x^2 + ſx + e^2 - \frac{1}{4}ſ^2$, ejus ſemidiameter erit $\infty \sqrt{e^2}$, hoc eſt $\infty \sqrt{\frac{1}{4}r^2 - \frac{1}{2}r + \frac{1}{4} + \frac{1}{4}ſ^2}$, ſcilicet $\infty \sqrt{\frac{1}{4}b^2 + \frac{1}{4}ſ^2}$, major pars diametri, qui per parabolæ axim ſuprà verticē productum orthogonaliter tranſit $\infty \sqrt{e^2} + \frac{1}{2}ſ$, minor $\infty \sqrt{e^2} - \frac{1}{2}ſ$, & ordinata ad diametrum $\infty \sqrt{\frac{1}{4}r^2 - \frac{1}{2}r + \frac{1}{4}}$, nimirum $\infty \frac{1}{2}r - \frac{1}{2}$, hoc eſt $\infty \frac{1}{2}b$.

Et hæc eſt Analyſis propoſitæ æquationis.

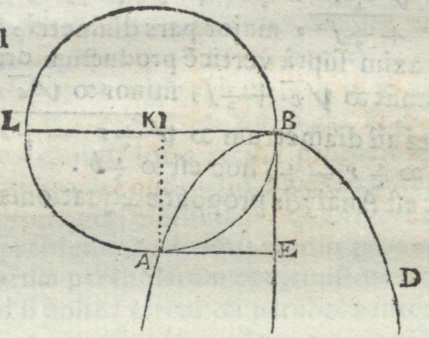
I



II



III



Deſcribatur pro conſtructione parabola ABD Fig. 1. cujus parameter *a*, ſive unitas. Sumatur portio axis BQ ſuprà verticem $\propto \tfrac{1}{2} b$, hoc eſt $\propto \overline{\tfrac{1}{2} r - \tfrac{1}{2}}$. Ducatur ipſi orthogonalis QK $\propto \tfrac{1}{2} s$, & producatur in rectum, donec KL ſit $\propto \sqrt{e^2}$, ſive $\propto \sqrt{\tfrac{1}{4} b^2 + \tfrac{1}{4} s^2}$, ideſt $\propto \sqrt{\tfrac{1}{4} r^2 - \tfrac{1}{2} r + \tfrac{1}{4} + \tfrac{1}{4} s^2}$, ſcilicet æqualis KB, nimirum lateri ex aggregato quadratorum BQ, QK. Conſtituatur ea circuli ſemi-

midiameter, cujus centrum K; dico hunc circulum parabolam ſecare in puncto A, (præter verticem, ſimiliter in circumferentia exiſtentem) ex quo ſi ducatur ad axim ordinata AE, hæc quæſitam radicem repræſentabit.

Et hæc eſt illa radix, ad quam eruendam Renatus Des-Cartes amplitudinis ſuæ methodi oblitus, confugit ad Cardani regulam Scipioni Ferreo Bononienſi tributam, ſed re verâ à Nicolao Tartalea Brixienſi inventam.

At ſi r minor eſſet quàm a, deberet ſumi $-b \propto \overline{r-a}$, & per additionem quadrati ex ipſius dimidio utrique parti æquationis, fiet æquatio ad circulum $-x^2 + \text{ſ}x + \frac{1}{4}b^2 \propto y^2 - by + \frac{1}{4}b^2$, cujus centrum erit in orthogonali ducta ex axis *Fig. II.* puncto Q infrà verticem, ab eodem diſtanti per $\frac{1}{2}b$, ſive per $-\frac{1}{2}r + \frac{1}{2}$.

Si verò eſſet æqualis a, invenietur, ut diximus, *Fig. III.* ad circulm æquatio $-x^2 + \text{ſ}x \propto y^2$, & ejus centrum erit in orthogonali ducta è verticis puncto B, ut ex conſtructionibus patet.

Idem quoque obſervare licet in cæteris æquationum trium, & quatuor graduum conſtructionibus, nam quando quantitas cognita ſecundi termini in ea parte, ubi eſt y^2, afficitur ſigno $+$, axis vel diameter parabolæ ultrà verticis punctum eſt producendus, & portio ipſius ſuprà verticem, ſumenda æqualis dimidio quantitatis notæ, quando verò afficitur ſigno $-$, citrà verticem in axe, vel diametro ſumenda eſt eadem portio, & per ipſius terminum, ſive ultrà, ſive citrà, diameter circuli ducendus; ſi autem ſecundus terminus abfuerit, tunc per verticem parabolæ diameter circuli tran-

ſibit

ſibit, & hoc ex ipſis æquationum constructionibus clarius conſtabit.

Sed progrediamur ad demonſtrationem poſitæ conſtructionis.

Quia MK, ſive KL eſt $\propto \sqrt{e^2}$, ſive $\propto \sqrt{\frac{1}{4}b^2 + \frac{1}{4}ſ^2}$, hoc eſt $\propto \sqrt{\frac{1}{4}r^2 - \frac{1}{2}r + \frac{1}{4} + \frac{1}{4}ſ^2}$, KQ $\propto \frac{1}{2}ſ$, & AE, ſive IQ $\propto x$, erit MI $\propto \sqrt{e^2} - \frac{1}{2}ſ + x$, hoc eſt $\propto \sqrt{\frac{1}{4}b^2 + \frac{1}{4}ſ^2} - \frac{1}{2}ſ + x$, nimirum $\propto \sqrt{\frac{1}{4}r^2 - \frac{1}{2}r + \frac{1}{4} + \frac{1}{4}ſ^2} - \frac{1}{2}ſ + x$, ac proinde IL $\propto \sqrt{e^2} + \frac{1}{2}ſ - x$, ſcilicet $\propto \sqrt{\frac{1}{4}b^2 + \frac{1}{4}ſ^2} + \frac{1}{2}ſ - x$, hoc eſt $\propto \sqrt{\frac{1}{4}r^2 - \frac{1}{2}r + \frac{1}{4} + \frac{1}{4}ſ^2} + \frac{1}{2}ſ - x$. Rurſus, quia AE $\propto x$, erit ex parabolæ proprietate, portio axis EB $\propto x^2$; ſi igitur huic axis portioni addatur portio ſuprà verticem BQ $\propto \frac{1}{2}b$, ſcilicet $\propto \frac{1}{2}r - \frac{1}{2}$, erit tota EQ, ſive AI $\propto x^2 + \frac{1}{2}b$, ſcilicet $\propto x^2 + \frac{1}{2}r - \frac{1}{2}$. Cùm igitur propter circulum, quadratum ex AI ſit æquale rectangulo ex MI in IL,

erit $x^4 + bx^2 + \frac{1}{4}b^2 \propto -x^2 + ſx + e^2 - \frac{1}{4}ſ^2$,

ſcilicet $x^4 + bx^2 + \frac{1}{4}b^2 \propto -x^2 + ſx + \frac{1}{4}b^2$,

$$\text{idest}\quad \begin{array}{lll} x^4 + rx^2 & + \frac{1}{4}r^2 \propto -x^2 + ſx & + \frac{1}{4}r^2 \\ - 1 & - \frac{1}{2}r & - \frac{1}{2}r \\ & + \frac{1}{4} & + \frac{1}{4} \end{array}$$

& deletis cõmunibus quantitatibus, emerget æquatio $x^4 + rx^2 \propto ſx$, hoc eſt $x^3 \propto * - rx + ſ$. Q.E.D.

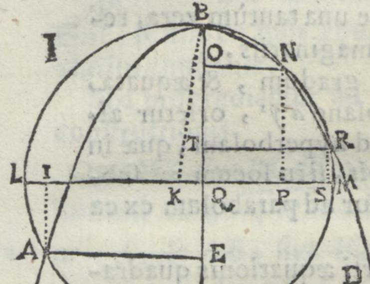
Proponatur nunc æquatio $x^3 \propto * + rx + ſ$, de tribus explicabilis radicibus, duabus falſis, & una

una vera falſis æquali, vel de una tantùm vera, reliquis duabus exiſtentibus imaginariis.

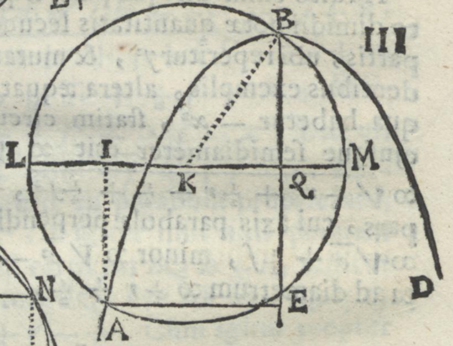
Elevata hac ad quartum gradum, & æquata utraque ejus parte plano-plano a^2y^2, orietur altera ad parabolam, altera ad hyperbolam, quæ in tertiam ad circulum tranſibit, ſi in locum x^2 ſubrogetur ejus valor ay, & prior ad parabolam ex ea auferatur.

Addito enim utrique parti æquationis quadrato dimidii notæ quantitatis ſecundi termini illius partis, ubi reperitur y^2, & mutata, ut in præcedentibus exemplis, altera æquationis parte, in qua habetur $-x^2$, ſtatim circulus apparebit, ejuſque ſemidiameter erit $\infty \sqrt{e^2}$, hoc eſt $\infty \sqrt{\frac{1}{4}r^2 + \frac{1}{2}r + \frac{1}{4} + \frac{1}{4}ſ^2}$, major diametri pars, cui axis parabolæ perpendiculariter inſiſtit $\infty \sqrt{e^2} + \frac{1}{2}ſ$, minor $\infty \sqrt{e^2} - \frac{1}{2}ſ$, & ordinata ad diametrum $\infty \frac{1}{2}r + \frac{1}{2}$.

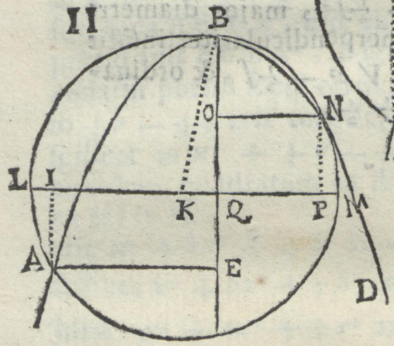
I



III



II



Deſcribatur pro conſtructione ex hac reſolutione deducta, quæ in omnibus cum Carteſiana congruit, parabola, cujus parameter a, ſive unitas, ſumatur portio axis BQ infrà verticem $\infty \frac{1}{2}r + \frac{1}{2}$, ducaturque ex puncto Q orthogonalis QK $\infty \frac{1}{2}ſ$, quæ producatur uſque ad L, ita ut KL ſit $\infty \sqrt{\frac{1}{4}r^2 + \frac{1}{2}r + \frac{1}{4} + \frac{1}{4}ſ^2}$; poſtmodum centro

tro K, & intervallo KL deſcribatur circulus, hic per parabolæ verticem tranſibit, & in tribus præterea punctis A, N, R, eam ſecabit, vel in duobus Fig. I. Fig. II. A, & N, quod accidit cùm falſæ radices inter ſe æquantur (tunc enim puncta N, & R congruunt, & circulus ibi parabolam tangit) vel demum in unico puncto A, ſi loco falſarum radicum, imaginariæ Fig. III. exiſtant; quod mirum eſt non vidiſſe Carteſium, qui pro veræ radicis explicatione ad Cardani regulam eò caſu confugit.

Si ergò ex his punctis ordinatæ ducantur, erunt hæ propoſitæ æquationis radices. AE vera, & NO, RT: vel NO bis ſumpta, falſæ.

Hæc tamen æquatio tunc erit de tribus explicabilis radicibus, cùm cubus tertiæ partis quantitatis notæ penultimi termini major eſt quadrato ex dimidio ultimi, nimirum $\frac{1}{27} r^3$ major quàm $\frac{1}{4} ſ^2$, vel $\frac{4}{27} r^3$, quàm $ſ^2$, ſive $\sqrt{\frac{4}{27} r^3}$ major $ſ$; nam ſi minor eſſet, de unica tantùm vera radice foret explicabilis, & reliquæ duæ imaginariæ exiſterent; ſi verò æqualis, falſæ radices inter ſe tunc æquarentur, & parabolam circulus in altero puncto ſecaret, in altero verò contingeret.

Id autem oritur ex natura æquationis, plures habentis radices, ex qua patet notam quantitatem ſecundi termini æqualem eſſe ſummæ radicum, tertii aggregato productorum ex ſingulis binis, quarti ſummæ productorum ex ſingulis ternis, atque ita deinceps. Demum ultimum terminum æqualem producto ex radicibus omnibus.

Quod jàm explicavimus habetur etiam ex doctrina de Maximis, & Minimis, quam nuper

Antonius Monfortius, vir omni literarum genere ornatiſſimus, in ſuo de Problematum Determinatione tractatu, facili, & nova methodo illuſtravit; per illam enim conſtat, ſi quantitas ſecetur in duas partes, quarum altera ſit dupla alterius, maximum ſolidum eſſe productum ex quadrato majoris in minorem; propterea $\frac{2}{3} r$ in $\sqrt{\frac{1}{3} r}$, ſcilicet $\sqrt{\frac{4}{27} r^3}$ maximum eſſe, ac proinde ſuperari non debere ab ultimo termino $ſ$, hoc eſt àproducto ex tribus propoſitæ æquationis radicibus.

Supereſt nunc demonſtrandum veram propoſitæ æquationis radicem eſſe AE, falſas NO, & RT; eaſque veræ æquales; ſed quia facilè hoc habetur ex præcedentibus exemplis, & ex demonſtratis per Franciſcum à Schooten, ad aliam progredimur æquationem $x^3 \infty * + rx - ſ$, de tribus ſimiliter explicabilem radicibus, una falſa, & duabus veris, falſæ æqualibus, vel de una tantum falſa, reliquis exiſtentibus imaginarijs.

In cōment. Carteſ. Geom.

Hæc ortum ſuum habet ex problemate de diviſione anguli, ſive arcûs circuli in tres æquales partes, & poſita a, ſive unitate pro radio, r pro 3 nota quantitate tertii termini, $ſ$ pro ſubtenſa arcûs dati, & x pro ſubtenſa arcûs quæſiti, habebitur ejus conſtructio eadem fermè ratione, ac præcedens, & cum Carteſiana in omnibus ſimiliter conveniet, non aliter, ac conſtructio problematis de duabus medio loco proportionalibus.

Neque ſolum quæ obſervavimus in præcedenti conſtructione, locum hìc habent, ſed nulla diſcrepantia inter illam, & hanc deprehenditur,

nisi

niſi quòd radix, quæ ibi eſt vera, falſa hìc reperitur, & quæ illic falſæ, veræ hìc habentur.

Hoc autem indicare videtur ſignum $-ſ$ diverſum ab illo in præcedenti $+ſ$, undè fortè Carteſio innotuit, veras radices æquationum, in quibus ultimus terminus afficitur ſigno $+$, eſſe ordinatas ductas ex parabolæ parte, ubi eſt circuli centrum; æquationum verò in quibus ultimus terminus reperitur affectus ſigno $-$, veras eſſe ordinatas ex altera parte ductas, falſas autem è contra.

Cæterùm ſi proponatur æquatio $x^3 \propto * - rx - ſ$, de unica tantum falſa radice explicabilis, reliquis imaginariis exiſtentibus; hæc abſque ope regulæ Cardani, eadem ratione conſtruetur, ac $x^3 \propto * - rx + ſ$, ſecundo loco propoſita; neque in alio hæc conſtructio ab illa diſcrepabit, niſi quòd ubi illic fuit inventa una radix vera, hìc deprehenditur una falſa.

Eadem ſimiliter ratione cõſtrui poterunt cæteræ æquationes trium, & quatuor dimenſionum, quarum formulas afferre iis, qui in hac re exerceri voluerint, ſuperfluum non exiſtimo.

$x^3 \propto ſ$

$x^3 \propto * - rx + ſ$
$x^3 \propto * + rx + ſ$
$x^3 \propto * + rx - ſ$
$x^3 \propto * - rx - ſ$

$x^3 \propto - qx^2 * + ſ$
$x^3 \propto + qx^2 * + ſ$
$x^3 \propto + qx^2 * - ſ$
$x^3 \propto - qx^2 * - ſ$

$x^3 \propto + qx^2 - rx + ſ$
$x^3 \propto - qx^2 + rx + ſ$
$x^3 \propto + qx^2 + rx + ſ$
$x^3 \propto - qx^2 - rx + ſ$
$x^3 \propto + qx^2 - rx - ſ$
$x^3 \propto - qx^2 + rx - ſ$
$x^3 \propto + qx^2 + rx - ſ$
$x^3 \propto - qx^2 - rx - ſ$

$x^4 \infty * * + rx - \text{ſ}$
$x^4 \infty * * + rx + \text{ſ}$
$x^4 \infty * * - rx + \text{ſ}$
$x^4 \infty * * - rx - \text{ſ}$

$x^4 \infty - px^3 * * + \text{ſ}$
$x^4 \infty + px^3 * * + \text{ſ}$
$x^4 \infty + px^3 * * - \text{ſ}$
$x^4 \infty - px^3 * * - \text{ſ}$

$x^4 \infty * - qx^2 * + \text{ſ}$
$x^4 \infty * + qx^2 * + \text{ſ}$
$x^4 \infty * + qx^2 * - \text{ſ}$
$x^4 \infty * - qx^2 * - \text{ſ}$

$x^4 \infty * + qx^2 - rx + \text{ſ}$
$x^4 \infty * - qx^2 + rx + \text{ſ}$
$x^4 \infty * + qx^2 + rx + \text{ſ}$
$x^4 \infty * - qx^2 - rx + \text{ſ}$
$x^4 \infty * + qx^2 - rx - \text{ſ}$
$x^4 \infty * - qx^2 + rx - \text{ſ}$
$x^4 \infty * + qx^2 + rx - \text{ſ}$
$x^4 \infty * - qx^2 - rx - \text{ſ}$

$x^4 \infty + px^3 - qx^2 * + \text{ſ}$
$x^4 \infty - px^3 + qx^2 * + \text{ſ}$
$x^4 \infty + px^3 + qx^2 * + \text{ſ}$
$x^4 \infty - px^3 - qx^2 * + \text{ſ}$
$x^4 \infty + px^3 - qx^2 * - \text{ſ}$
$x^4 \infty - px^3 + qx^2 * - \text{ſ}$
$x^4 \infty + px^3 + qx^2 * - \text{ſ}$
$x^4 \infty - px^3 - qx^2 * - \text{ſ}$

$x^4 \infty + px^3 * - rx + \text{ſ}$
$x^4 \infty - px^3 * + rx + \text{ſ}$
$x^4 \infty + px^3 * + rx + \text{ſ}$
$x^4 \infty - px^3 * - rx + \text{ſ}$
$x^4 \infty + px^3 * - rx - \text{ſ}$
$x^4 \infty - px^3 * + rx - \text{ſ}$
$x^4 \infty + px^3 * + rx - \text{ſ}$
$x^4 \infty - px^3 * - rx - \text{ſ}$

$x^4 \infty + px^3 - qx^2 + rx - \text{ſ}$
$x^4 \infty + px^3 - qx^2 - rx - \text{ſ}$
$x^4 \infty + px^3 + qx^2 + rx - \text{ſ}$
$x^4 \infty + px^3 + qx^2 - rx - \text{ſ}$
$x^4 \infty - px^3 - qx^2 + rx - \text{ſ}$
$x^4 \infty - px^3 + qx^2 + rx - \text{ſ}$
$x^4 \infty - px^3 + qx^2 - rx - \text{ſ}$
$x^4 \infty - px^3 - qx^2 - rx - \text{ſ}$
$x^4 \infty + px^3 - qx^2 + rx + \text{ſ}$
$x^4 \infty + px^3 - qx^2 - rx + \text{ſ}$
$x^4 \infty + px^3 + qx^2 + rx + \text{ſ}$
$x^4 \infty + px^3 + qx^2 - rx + \text{ſ}$
$x^4 \infty - px^3 - qx^2 + rx + \text{ſ}$
$x^4 \infty - px^3 - qx^2 - rx + \text{ſ}$
$x^4 \infty - px^3 + qx^2 + rx + \text{ſ}$
$x^4 \infty - px^3 + qx^2 - rx + \text{ſ}$

Præ-

Præterea ſi quis methodum hanc in planis æquationibus compoſitis experiri voluerit, non quidem inutilem inveniet, quamquam aliæ paratiores apud alios non deſint.

Sint formulæ planarum æquationum.

$x^2 \propto + px - q$
$x^2 \propto + px + q$
$x^2 \propto - px + q$
$x^2 \propto - px - q$

Ponatur conſtruenda prima formula $x^2 \propto px - q$, de duabus explicabilis veris radicibus.

Elevata hac æquatione ad quartum gradum, atque addito utrique parti quadrato ex latere $\frac{1}{2}px$, ut una æquationis pars conſtituat quadratum, habebitur $x^4 - px^3 + \frac{1}{4}p^2x^2 \propto \begin{array}{l} -\ qx^2 \\ +\frac{1}{4}p^2 \end{array}$

Sumpto deinde plano-plano a^2y^2, invenientur æquationes $x^4 - px^3 + \frac{1}{4}p^2x^2 \propto a^2y^2$, ſcilicet $x^2 - \frac{1}{2}px \propto ay$ ad parabolam, quæ ſi addatur propoſitæ æquationi $x^2 \propto px - q$, hoc eſt $px - x^2 \propto q$, mutabitur in $\frac{1}{2}px \propto ay + q$ ad rectam lineam, & $\begin{array}{l} -\ qx^2 \\ +\frac{1}{4}p^2 \end{array} \propto y^2$ ad hyperbolam, ſi quadratum dimidii quantitatis notæ ſecundi termini majus ſit ultimo termino, ſcilicet $\frac{1}{4}p^2$ majus q; nam ſi minus foret, impoſſibilis eſſet conſtructio, & pro veris radicibus imaginariæ haberentur; ſi verò æquale, ultimus terminus deſtrueretur, & ex plana, ſimplex fieret æquatio. Atque hoc patet, non ſolum ex doctrina de Maximis ab Euclide in Elementis 28. ſexti. indicata, per quam conſtat quadratum prædictum maximum eſſe rectangulorum omnium, quæ ex ſectione ipſius quantitatis p, hoc eſt ſummæ radicum

in duas partes oriri poſſunt, ſed etiã ex ipſa æquatione, $\begin{array}{l} -\ q \\ +\frac{1}{4}p^2 \end{array} x^2 \propto y^2$, nam ſi q major foret, vel æquale $\frac{1}{4}p^2$, quantitas y^2, vel nihilo minor eſſet, vel nihilo æquaretur, quod ἀδυναμίαν arguit.

Sed ut ex hyperbola ad circulum tranſitus fiat, eodem, quo ſuprà, modo ultrà procedendo, habebimus æquationem

$$\begin{array}{llllll} -x^2 & +\frac{1}{2}px & +\frac{1}{4}q^2 & \propto y^2 & -\frac{1}{4}p^2y & +\frac{1}{4}q^2, \\ & -\frac{1}{2}qp & -\frac{1}{8}p^2q & + & q & -\frac{1}{8}p^2q \\ & +\frac{1}{8}p^3 & -\frac{1}{2}q & = & a & -\frac{1}{2}q \\ & & +\frac{1}{64}p^4 & & & +\frac{1}{64}p^4 \\ & & +\frac{1}{8}p^2 & & & +\frac{1}{8}p^2 \\ & & +\frac{1}{4} & & & +\frac{1}{4} \end{array}$$

& mutata illius parte, ubi habetur $-x^2$ in

$$\begin{array}{llllll} -x^2 & +\frac{1}{2}px & +\ e^2 & \propto y^2 & -\frac{1}{4}p^2y & +\frac{1}{4}q^2, \\ & -\frac{1}{2}qp & -\frac{1}{256}p^6 & + & q & -\frac{1}{8}p^2q \\ & +\frac{1}{8}p^3 & -\frac{1}{32}p^4 & - & a & -\frac{1}{2}q \\ & & +\frac{1}{32}qp^4 & & & +\frac{1}{64}p^4 \\ & & -\frac{1}{16}p^2 & & & +\frac{1}{8}p^2 \\ & & +\frac{1}{8}qp^2 & & & +\frac{1}{4} \\ & & -\frac{1}{16}q^2p^2 & & & \end{array}$$

ejus ſemidiameter erit $\propto \sqrt{e^2}$, nimirum $\propto \sqrt{\frac{1}{4}q^2 - \frac{1}{8}p^2q - \frac{1}{2}q + \frac{1}{64}p^4 + \frac{1}{8}p^2 + \frac{1}{4} + \frac{1}{256}p^6 + \frac{1}{32}p^4 - \frac{1}{32}qp^4 + \frac{1}{16}p^2 - \frac{1}{8}qp^2 + \frac{1}{16}q^2p^2}$, major diametri pars, $\propto \sqrt{e^2} + \frac{1}{16}p^3 + \frac{1}{4}p - \frac{1}{4}qp$, minor $\propto \sqrt{e^2} - \frac{1}{16}p^3 - \frac{1}{4}p + \frac{1}{4}qp$;

&

& ordinata ad diametrum $\infty \sqrt{\frac{1}{4}q^2 - \frac{1}{8}p^2q - \frac{1}{2}q + \frac{1}{64}p^4 + \frac{1}{8}p^2 + \frac{1}{4}}$, hoc eſt $\infty \frac{1}{2}q - \frac{1}{8}p^2 - \frac{1}{2}$. Atque ita conſtructio propoſitæ æquationis rectæ, & circuli beneficio habebitur.

Progrediamur jam ad conſtructionem æquationum altiorum graduum.

Proponatur conſtruenda æquatio ſex dimenſionum, in qua nullus terminus deficiat.

Primò ad quintum gradum deprimatur, hoc eſt ad

$$x^5 - px^4 + qx^3 - rx^2 + \text{ſ}x - t \infty 0$$

Deinde ad ſextum gradum revocetur, nimirum ad

$x^6 - px^5 + qx^4 - rx^3 + \text{ſ}x^2 - tx \infty 0$, ſcilicet $x^6 \infty px^5 - qx^4 + rx^3 - \text{ſ}x^2 + tx$.

Poſtmodum, ut evaneſcat ſecundus æquationis terminus, formetur quadratum ex latere $x^3 - \frac{1}{2}px^2$, ideſt $x^6 - px^5 + \frac{1}{4}p^2x^4$, addaturque utrique parti æquationis portio $-px^5 + \frac{1}{4}p^2x^4$, & habebitur

$$x^6 - px^5 + \frac{1}{4}p^2x^4 \infty \begin{array}{l} px^5 \\ -p \end{array} \begin{array}{l} -\,qx^4 \\ +\frac{1}{4}p^2 \end{array} + rx^3 - \text{ſ}x^2 + tx,$$

ſive $x^6 - px^5 + \frac{1}{4}p^2x^4 \infty \begin{array}{l} -\,qx^4 \\ +\frac{1}{4}p^2 \end{array} + rx^3 - \text{ſ}x^2 + tx.$

Demum utrique ejus parti fiat æqualis poteſtas $a^2y^2x^2$, ſumpta a pro unitate, & duæ invenientur æquationes, $x^6 - px^5 + \frac{1}{4}p^2x^4 \infty a^2y^2x^2$, ſcilicet $x^3 - \frac{1}{2}px^2 \infty ayx$, ſive $x^2 - \frac{1}{2}px \infty ay$ ad planam parabolam, & $\begin{array}{l} -\,qx^4 \\ +\frac{1}{4}p^2 \end{array} + rx^3 - \text{ſ}x^2 + tx \infty a^2y^2x^2$,

ſci-

$$\text{ſcilicet}\quad \begin{array}{l} -\,qx^3 \\ +\frac{1}{4}p^2 \end{array} + rx^2 - sx + t \infty a^2y^2x\,,$$

$$\text{nimirum}\quad \begin{array}{l} -\,aqx^3 \\ +\frac{1}{4}p^2 \end{array} + a^2rx^2 - a^3sx + a^4t \infty a^2y^2x\,,$$

$$\text{hoc eſt}\quad \begin{array}{l} -\frac{q}{a}x^3 \\ +\frac{1}{4}\frac{p^2}{a^2} \end{array} + rx^2 - asx + a^2t \infty y^2x$$

ad curvam tertii gradus, ſive cubicam hyperbolam.

Hac igitur methodo, beneficio planæ parabolæ, & cubicæ hyperbolæ æquatio propoſita facilè conſtrui poterit.

Sed ſi pro quadrato ex latere $x^3 - \frac{1}{2}px^2$, fingatur quadratum ex altero latere $x^3 - \frac{1}{2}px^2 + \begin{array}{l} \frac{1}{2}qx, \\ -\frac{1}{8}p^2 \end{array}$ regula mox explicanda, invento, ut ſecundus, & tertius terminus propoſitæ æquationis deſtruantur, eorumdem graduum curvæ pro conſtruenda æquatione reperientur.

Sit igitur quadratum ex præfato latere

$$x^6 - px^5 + qx^4 \begin{array}{l} -\frac{1}{2}pqx^3 \\ +\frac{1}{8}p^3 \\ \end{array} \begin{array}{l} +\frac{1}{4}q^2x^2. \\ -\frac{1}{8}qp^2 \\ +\frac{1}{64}p^4 \end{array}$$

Addatur utrique parti æquationis portio ipſius

$$-px^5 + qx^4 \begin{array}{l} -\frac{1}{2}pqx^3 \\ +\frac{1}{8}p^3 \\ \end{array} \begin{array}{l} +\frac{1}{4}q^2x^2 \\ -\frac{1}{8}qp^2 \\ +\frac{1}{64}p^4 \end{array}$$

, & habebitur

alte-

altera æquatio
$$\begin{array}{lllll} x^6 & -px^5 & +qx^4 & -\frac{1}{2}pqx^3 & +\frac{1}{4}q^2x^2 \\ & & & +\frac{1}{8}p^3 & -\frac{1}{8}qp^2 \\ & & & & +\frac{1}{64}p^4 \end{array}$$

$$\begin{array}{llllll} \infty & px^5 & -qx^4 & +rx^3 & -\text{ſ}x^2 & +tx, \\ & -p & +q & -\frac{1}{2}pq & +\frac{1}{4}q^2 & \\ & & & +\frac{1}{8}p^3 & -\frac{1}{8}qp^2 & \\ & & & & +\frac{1}{64}p^4 & \end{array}$$

nimirum
$$\begin{array}{lllll} x^6 & -px^5 & +qx^4 & -\frac{1}{2}pqx^3 & +\frac{1}{4}q^2x^2 \\ & & & +\frac{1}{8}p^3 & -\frac{1}{8}qp^2 \\ & & & & +\frac{1}{64}p^4 \end{array}$$

$$\begin{array}{llll} \infty & rx^3 & -\text{ſ}x^2 & +tx. \\ & -\frac{1}{2}pq & +\frac{1}{4}q^2 & \\ & +\frac{1}{8}p^3 & -\frac{1}{8}qp^2 & \\ & & +\frac{1}{64}p^4 & \end{array}$$

Æquetur deinde utrique parti poteſtas $a^2y^2x^2$; & invenientur æquationes,

$$\begin{array}{llllll} x^5 & -px^5 & +qx^4 & -\frac{1}{2}pqx^3 & +\frac{1}{4}q^2x^4 & \infty\ a^2y^2x^2, \\ & & & +\frac{1}{8}p^3 & -\frac{1}{8}qp^2 & \\ & & & & +\frac{1}{64}p^4 & \end{array}$$

ſcilicet
$$\begin{array}{lllll} x^3 & -\frac{1}{2}px^2 & +\frac{1}{2}qx & \infty\ ayx; \\ & & -\frac{1}{8}p^2 & \end{array}$$

ſive
$$\begin{array}{llll} x^2 & -\frac{1}{2}px & +\frac{1}{2}q & \infty\ ay, \\ & & -\frac{1}{8}p^2 & \end{array}$$
ad planam parabolam,

&
$$\begin{array}{lll} rx^3 - & ſx^2 & + tx \propto a^2y^2x^2, \\ -\frac{1}{2}pq & +\frac{1}{4}q^2 & \\ +\frac{1}{8}p^3 & -\frac{1}{8}qp^2 & \\ & +\frac{1}{64}p^4 & \end{array}$$

ideſt
$$\begin{array}{lll} rx^2 - & ſx & + t \propto a^2y^2x; \\ -\frac{1}{2}pq & +\frac{1}{4}q^2 & \\ +\frac{1}{8}p^3 & -\frac{1}{8}qp^2 & \\ & +\frac{1}{64}p^4 & \end{array}$$

nimirum
$$\begin{array}{lll} a^2rx^2 - & a^3ſx & + a^4t \propto a^2y^2x, \\ -\frac{1}{2}apq & +\frac{1}{4}a^2q^2 & \\ +\frac{1}{8}p^3 & -\frac{1}{8}aqp^2 & \\ & +\frac{1}{64}p^4 & \end{array}$$

hoc eſt
$$\begin{array}{lll} rx^2 - & aſx & + a^2t \propto y^2x; \\ -\frac{1}{2}\frac{pq}{a} & +\frac{1}{4}q^2 & \\ +\frac{1}{8}\frac{p^3}{a^2} & -\frac{1}{8}\frac{qp^2}{a} & \\ & +\frac{1}{64}\frac{p^4}{a^2} & \end{array}$$

ad curvam tertii gradus, ſive ſecundi generis: Demum ſi pro quadrato ex
$$\begin{array}{l} x^3 - \frac{1}{2}px^2 + \frac{1}{2}qx, \\ \qquad\qquad\qquad -\frac{1}{8}p^2 \end{array}$$
formetur quadratũ ex
$$\begin{array}{l} x^3 - \frac{1}{2}px^2 + \frac{1}{2}qx - \frac{1}{2}r, \\ \qquad\qquad\qquad -\frac{1}{8}p^2 + \frac{1}{4}pq \\ \qquad\qquad\qquad\qquad\qquad -\frac{1}{16}p^3 \end{array}$$

ut

ut tres æquationis termini $px^5 - qx^4 + rx^3$ evaneſcant, eorumdem graduum curvis æquationis conſtructio perficietur, verùm prior æquatio

$$\begin{aligned} x^3 - \tfrac{1}{2}px^2 + \tfrac{1}{2}qx - \tfrac{1}{2}\ r \infty a^2y, \\ - \tfrac{1}{8}p^2 + \tfrac{1}{4}pq \\ - \tfrac{1}{16}p^3 \end{aligned}$$

erit ad curvam tertii gradus, & poſterior

$$\begin{aligned} - \tfrac{ſ}{a}x^2 + \ tx + \tfrac{1}{4}r^2 \ \infty y^2, \\ + \tfrac{1}{4}\tfrac{q^2}{a^2} - \tfrac{1}{2}\tfrac{qr}{a} - \tfrac{1}{4}\tfrac{pqr}{a} \\ + \tfrac{1}{2}\tfrac{pr}{a^2} + \tfrac{1}{4}\tfrac{pq^2}{a^2} + \tfrac{1}{16}\tfrac{p^3r}{a^2} \\ - \tfrac{3}{8}\tfrac{qp_2}{a^3} + \tfrac{1}{8}\tfrac{p^2r}{a^2} + \tfrac{1}{16}\tfrac{p^2q_2}{a^2} \\ + \tfrac{5}{64}\tfrac{p_4}{a^4} - \tfrac{1}{8}\tfrac{qp^3}{a^3} - \tfrac{1}{32}\tfrac{qp_4}{a_3} \\ + \tfrac{1}{64}\tfrac{p^5}{a^4} + \tfrac{1}{256}\tfrac{p^6}{a^4} \end{aligned}$$

ad curvam ſecundi, ſumpta communi poteſtate a^4y^2; nam ſi ea foret $a^2y^2x^2$, invenietur prior æquatio

$$\begin{aligned} x^3 - \tfrac{1}{2}px^2 + \tfrac{1}{2}qx - \tfrac{1}{2}\ r \infty ayx, \\ - \tfrac{1}{8}p^2 + \tfrac{1}{4}pq \\ - \tfrac{1}{16}p^3 \end{aligned}$$

ad curvam tertii gradus, & poſterior, nimi-

rum

rum $- a\int x^2 + a^2tx + \frac{1}{4}a^2r^2 \infty y^2x^2$;

$$+\tfrac{1}{4}q^2 - \tfrac{1}{2}aqr - \tfrac{1}{4}apqr$$
$$+\tfrac{1}{2}pr + \tfrac{1}{4}pq^2 + \tfrac{1}{16}p^3r$$
$$-\tfrac{3}{8}\frac{qp^2}{a} + \tfrac{1}{8}p^2r + \tfrac{1}{16}p^2q^2$$
$$+\tfrac{5}{64}\frac{p^4}{a^2} - \tfrac{1}{8}\frac{qp^3}{a} - \tfrac{1}{32}\frac{qp^4}{a}$$
$$+\tfrac{1}{64}\frac{p^5}{a^2} + \tfrac{1}{256}\frac{p^6}{a^2}$$

ad curvam quarti, atque ita proprius pro conſtructione locus non habebitur.

Eadem ratione conſtrui poterunt cœteræ æquationes ſex dimenſionum.

Proponatur nunc æquatio quinque graduum, ſecundo carens termino.

Hæc methodo tradita faciliùs conſtruetur; etenim nulla tunc requiritur depreſſio, aut fictio quadrati ex aliquo latere, ſed tantùm ad parem gradum erit æquatio elevanda.

Sit ea igitur $x^5 * - 5x^3 * + 5x - q \infty 0$, quæ oritur ex problemate ſectionis anguli, ſive arcûs circuli in quinque æquales partes

Extollatur primò ad parem gradum,

ut fiat $x^6 * - 5x^4 * + 5x^2 - qx \infty 0$,

ſcilicet $x^6 \infty * + 5x^4 * - 5x^2 + qx$,

nimirum $x^6 \infty * + qx^4 * - \int x^2 + tx$, poſitis pro primo numero 5 litera q, pro altero numero 5 litera $\int$, & pro ſubtenſa arcûs dati, ſive ultimo termino q, litera t.

Ponatur deinde utraque ejus pars æqualis pote-

ſtati

ſtati $a^2y^2x^2$, & emergent æquationes $x^6 \infty a^2y^2x^2$; ſcilicet $x^3 \infty ayx$, nimirum $x^2 \infty ay$ ad planam parabolam, & $qx^4 - ſx^2 + tx \infty a^2y^2x^2$, hoc eſt $qx^3 - ſx + t \infty a^2y^2x$, ſcilicet $\frac{qx^3}{a} - aſx + a^2t \infty y^2x$, ad curvam tertii gradus, ſive cubicam hyperbolam.

Quòd ſi fingatur quadratum ex latere $x^3 - \frac{1}{2}qx$, & ut deſtruatur tertius terminus qx^4, utrique parti æquationis addatur portio ipſius $-qx^4 + \frac{1}{4}q^2x^2$, eorumdem graduum curvæ pro conſtructione invenientur.

Quoniam æquatio juxtà methodum deducta, eſt

$$x^6 - qx^4 + \tfrac{1}{4}q^2x^2 \infty \begin{array}{l} qx^4 \\ -q \end{array} \begin{array}{l} -ſx^2 \\ +\frac{1}{4}q^2 \end{array} + tx,$$

ſive $x^6 - qx^4 + \frac{1}{4}q^2x^2 \infty \begin{array}{l} -ſx^2 \\ +\frac{1}{4}q^2 \end{array} + tx$, ſumpta communi poteſtate $a_2y^2x^2$, emergent æquationes $x^6 - qx^4 + \frac{1}{4}q^2x^2 \infty a^2y^2x^2$, ideſt $x^3 - \frac{1}{2}qx \infty ayx$, hoc eſt $x^2 - \frac{1}{2}q \infty ay$, ad planam parabolam, & $\begin{array}{l} -ſx^2 \\ +\frac{1}{4}q^2 \end{array} + tx \infty a^2y^2x$, ſcilicet $\begin{array}{l} -ſx \\ +\frac{1}{4}q^2 \end{array} + t \infty a^2y^2x$, nimirum $\begin{array}{l} a^3ſx \\ +\frac{1}{4}a^2q^2 \end{array} + a^4t \infty a^2y^2x$, hoc eſt $\begin{array}{l} -aſx \\ +\frac{1}{4}q^2 \end{array} + a^2t \infty y^2x$ ad curvam tertii gradus.

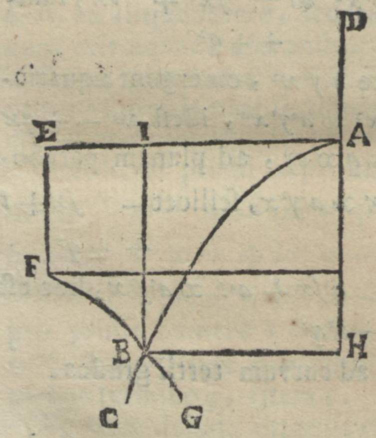
Sed

Sed ſi communis poteſtas eſſet a^4y^2, invenientur $x^3 - \frac{1}{2}qx \infty a^2y$ ad curvam tertii gradus,

$$\& \quad \begin{matrix} -\frac{s^2}{a} \\ +\frac{1}{4}\frac{q^2}{a^2} \end{matrix} x^2 + tx \infty y^2$$ ad curvam ſecundi.

Rurſus proponatur æquatio nullum habens interpoſitum terminum $x^5 \infty a^4p$, quæ oritur ex problemate de quatuor medijs proportionalibus inter datas a, & p inveniendis.

Exaltetur hæc ad parem gradum, & utraque ejus pars ponatur æqualis poteſtati $a^2y^2x^2$; emergẽt juxtà methodum æquationes $x^6 \infty a^2y^2x^2$, hoc eſt $x^3 \infty ayx$, ſive $x^2 \infty ay$ ad planam parabolam, & $a^4px \infty a^2y^2x^2$, ſcilicet $a^2p \infty y^2x$ ad hyperbolam ſecundi gradus, ex quibus conſtructio facillimè deducitur.



Deſcribatur plana parabola ABC, cujus parameter AD, ſive a prima ex datis; Ducatur deindè ex verticis pũcto A orthogonalis AE æqualis p alteri ex datis, atque ex E puncto EF parallela axi parabolæ, parametro æqualis; Demum, ex F puncto, Aſymptotis EA, AH, deſcri-

ſcribatur hyperbola ſecundi gradus, in qua datæ EF quadratum ad quadratum ex IB, ſive AH in eadem ſit ratione, ut indeterminata BH, ſive IA ad alteram ex datis EA. Et quoniam hæc hyperbola deſcriptam parabolam in puncto B ſecat, ſi ordinata BH fuerit ex eo ducta, erit illa propoſitæ æquationis radix, ac proinde prima ex quatuor mediis, quod ita demonſtratur.

Quia BH inventa eſt $\infty\ x$, erit per parabolam AH $\infty\ \frac{x^2}{a}$. Quia ſimiliter per deſcriptam hyperbolam, quadratum ex AH, ſive IB, ſcilicet $\frac{x^4}{a^2}$ ad quadratũ ex EF, ſive a^2 eandem habet rationem, quam recta AE, ſive p ad rectam BH $\infty\ x$, erit $\frac{x^5}{a^2} \infty\ a^2p$, hoc eſt $x^5 \infty\ a^4p$. Q. E. D.

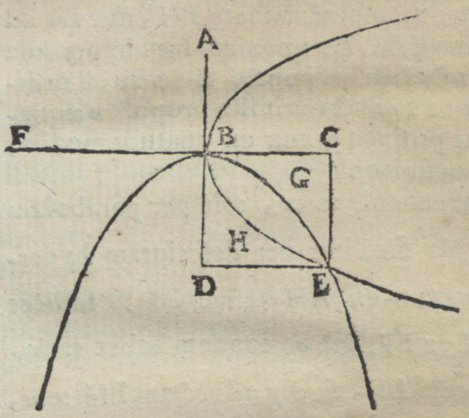
Aliter hoc demonſtrari poteſt ex æquationibus, deductis ab illa, quam primo loco conſtruendam propoſuimus.

Quia parabolæ parameter eſt $\infty\ a$, BH ordinata $\infty\ x$, & portio axis AH $\infty\ y$, erit propter eam $x^2 \infty\ ay$. Rurſus quia per deſcriptæ hyperbolæ proprietatem eſt y^2 ad a^2, ut p ad x, erit $y^2x \infty\ a^2p$. Q. E. D.

Atque hæc eſt propoſiti problematis conſtructio, quæ inveniri poterat abſque exaltatione æquationis ad ſextum gradum, ſumpta tantummodo poteſtate a^2y^2x, utrique ejus parti æquali.

Quod ſi, elevata ad ſextum gradum æquatione, communis poteſtas eſſet a^4y^2, orietur conſtructio à præcedenti diverſa, duarum ope parabolarum,

planæ nimirum , & cubicæ , quarum æquationes erunt $px \infty y^2$, & $x^3 \infty a^2y$.



Poſitis igitur ad rectos angulos rectis AB ∞a , & BF ∞p , ſi puncto B communi vertice, deſcribatur plana parabola BGE , cujus parameter BF , & cubica BHE ejus proprietatis , ut cubus ordinatæ æquetur ſolido ex quadrato parametri AB in axis portionem ab ordinata interceptam ; ſi poſtea ducatur ordinata EC ex puncto E, in quo hæ parabolæ ſe interſecant , erit ea prima ex quæſitis mediis , ac proinde radix propoſitæ æquationis , quod ita demonſtratur.

Quia CE eſt ∞x , erit propter parabolam cubicam BC , ſive DE ordinata ad planam $\infty \frac{x^3}{a^2}$, ac ideo per proprietatem ipſius $\frac{x^6}{a^4} \infty px$, hoc eſt $x^6 \infty a^4px$, nimirum $x^5 \infty a^4 p$. Q. E. D.

Ex his manifeſtum fit methodum , qua utimur , non ſolum in conſtructionibus æquationum trium , & quatuor dimenſionum , ſed etiam in aliis quinque , ſex , & altiorum graduum faciliorem eſſe ,

illa, quam adhibet Carteſius pro conſtruendis ſimilibus æquationibus, atque ſimpliciorem eâ, quam Fermatius affert; Cùm enim duas omninò tertii gradus curvas pro hiſce æquationibus hic Author requirat, unicam tantùm nobis noſtra methodus ſuggerit, quæ cum altera ſecundi gradus ſe interſecans, rem feliciter perficit.

In fine ſuæ Geometriæ.

In Diſſert. contrà Carteſ. part. 2.

Demum ut pateat hujus methodi amplitudo, proponatur conſtruenda æquatio octo dimenſionum, in qua omnes extent termini.

Depreſſa hac ad ſeptimum, & rurſus ad octavum gradum revocata, ut pares habeat æquatio dimenſiones, erit

$$x^8 - ox^7 + px^6 - qx^5 + rx^4 - \text{ſ}x^3 + tx^2 - vx \infty 0,$$

ſive $x^8 \infty ox^7 - px^6 + qx^5 - rx^4 + \text{ſ}x^3 - tx^2 + vx$.

Poſtmodum effícto quadrato ex latere

$$\begin{array}{llll} x^4 - \frac{1}{2}ox^3 & + \frac{1}{2}px^2 & + \frac{1}{4}opx, \\ & - \frac{1}{8}o^2 & - \frac{1}{16}o^3 \\ & & - \frac{1}{2}q \end{array}$$

ſcilicet

$$\begin{array}{lllll} x^8 - ox^7 + px^6 - qx^5 & + \frac{1}{4}p^2x^4 & + \frac{1}{4}op^2x^3 & + \frac{1}{4}q^2x^2, \\ & - \frac{3}{8}o^2p & - \frac{1}{8}o^3p & + \frac{1}{16}o^2p^2 \\ & + \frac{5}{64}o^4 & - \frac{1}{2}pq & - \frac{1}{32}o^4p \\ & + \frac{1}{2}oq & + \frac{1}{64}o^5 & - \frac{1}{4}opq \\ & & + \frac{1}{8}o^2q & + \frac{1}{256}o^6 \\ & & & + \frac{1}{16}o^3q \end{array}$$

ut deſtruantur tres termini $ox^7 - px^6 + qx^5$; ſi utrique parti æquationis addatur portio ipſius

$$\begin{array}{llll}
-ox^7+px^6-qx^5 & +\frac{1}{4}p^2x^4 & +\frac{1}{4}op^2x^3 & +\frac{1}{4}q^2x^2; \\
 & -\frac{3}{8}o^2p & -\frac{1}{8}o^3p & +\frac{1}{16}o^2p^2 \\
 & +\frac{5}{64}o^4 & -\frac{1}{2}pq & -\frac{1}{32}o^4p \\
 & +\frac{1}{2}oq & +\frac{1}{64}a^5 & -\frac{1}{4}opq \\
 & & +\frac{1}{8}o^2q & +\frac{1}{256}o^6 \\
 & & & +\frac{1}{16}o^3q
\end{array}$$

habebitur

$$\begin{array}{llll}
x^8-ox^7+px^6-qx^5 & +\frac{1}{4}p^2x^4 & +\frac{1}{4}op^2x^3 & +\frac{1}{4}q^2x^2; \\
 & -\frac{3}{8}o^2p & -\frac{1}{8}o^3p & +\frac{1}{16}o^2p^2 \\
 & +\frac{5}{64}o^4 & -\frac{1}{2}pq & -\frac{1}{32}o^4p \\
 & +\frac{1}{2}oq & +\frac{1}{64}o^5 & -\frac{1}{4}opq \\
 & & +\frac{1}{8}o^2q & +\frac{1}{256}o^6 \\
 & & & +\frac{1}{16}o^3q
\end{array}$$

$$\begin{array}{llll}
\infty - & rx^4 + & ſx^3 = & tx^2 + vx; \\
 & +\frac{1}{4}p^2 & +\frac{1}{4}op^2 & +\frac{1}{4}q^2 \\
 & -\frac{3}{8}o^2p & -\frac{1}{8}o^3p & +\frac{1}{16}o^2p^2 \\
 & +\frac{5}{64}o^4 & -\frac{1}{2}pq & -\frac{1}{32}o^4p \\
 & +\frac{1}{2}oq & +\frac{1}{64}o^5 & -\frac{1}{4}opq \\
 & & +\frac{1}{8}o^2q & +\frac{1}{256}o^6 \\
 & & & +\frac{1}{16}o^3q
\end{array}$$

Si deinde ſumpta a pro unitate; ponatur utrique parti æqualis poteſtas $a^4y^2x^2$, fient æquationes

x^8

$$\begin{array}{lllll}
x^8 - ox^7 + px^6 - qx^5 & + \frac{1}{4}p^2x^4 & + op^2x^3 & + q^2x^2 \propto a^4y^2x^2, \\
 & - \frac{3}{8}o^2p & - \frac{1}{8}o^3p & + \frac{1}{16}o^2p^2 \\
 & + \frac{5}{64}o^4 & - \frac{1}{2}pq & - \frac{1}{32}o^4p \\
 & + \frac{1}{2}oq & + \frac{1}{64}o^5 & - \frac{1}{4}opq \\
 & & + \frac{1}{8}o^2q & + \frac{1}{256}o^6 \\
 & & & + \frac{1}{16}o^3q
\end{array}$$

$$\begin{array}{llll}
\text{ſcilicet } x^4 - \frac{1}{2}ox^3 & + \frac{1}{2}px^2 & + \frac{1}{4}opx \propto a^2yx, \\
 & - \frac{1}{8}o^2 & - \frac{1}{16}o^3 \\
 & & - \frac{1}{2}q
\end{array}$$

$$\begin{array}{llll}
\text{hoc eſt } x^3 - \frac{1}{2}ox^2 & + \frac{1}{2}px & + \frac{1}{4}op \propto a^2y \\
 & - \frac{1}{8}o^2 & - \frac{1}{16}o^3 \\
 & & - \frac{1}{2}q
\end{array}$$

ad curvam tertii gradus.

$$\begin{array}{llll}
\text{Et } - rx^4 & + ſx^3 & - tx^2 + vx \propto a^4y^2x^2, \\
+ \frac{1}{4}p^2 & + \frac{1}{4}op^2 & + \frac{1}{4}q^2 \\
- \frac{3}{8}o^2p & - \frac{1}{8}o^3p & + \frac{1}{16}o^2p^2 \\
+ \frac{5}{64}o^4 & - \frac{1}{2}pq & - \frac{1}{32}o^4p \\
+ \frac{1}{2}oq & + \frac{1}{64}o^5 & - \frac{1}{4}opq \\
 & + \frac{1}{8}o^2q & + \frac{1}{256}o^5 \\
 & & + \frac{1}{16}o^3q
\end{array}$$

$$\begin{array}{llll}
\text{ideſt } - a^3rx^4 & + a^4ſx^3 & - a^5tx^2 + a^6rx \propto a^4y^2x^2, \\
+ \frac{1}{4}a^2p^2 & + \frac{1}{4}a^2op^2 & + \frac{1}{4}a^4q^2 \\
- \frac{3}{8}ao^2p & - \frac{1}{8}ao^3p & + \frac{1}{16}a^2o^2p^2 \\
+ \frac{5}{64}o^4 & - \frac{1}{2}a^3pq & - \frac{1}{32}ao^4p \\
+ \frac{1}{2}a^2oq & + \frac{1}{64}o^5 & - \frac{1}{4}a^3opq \\
 & + \frac{1}{8}a^2o^2q & + \frac{1}{256}o^6 \\
 & & + \frac{1}{16}a^2o^3q
\end{array}$$

Ill.

$$\begin{array}{lllll}
\text{nimirũ} & -\frac{r}{a}x^4 + & s x^3 - & a t x^2 + a^2 v x \propto y^2 x^2; \\
& +\frac{1}{4}\frac{p^2}{a^2} & +\frac{1}{4}\frac{op^2}{a^2} & +\frac{1}{4}q^2 \\
& -\frac{3}{8}\frac{o^2p}{a^3} & -\frac{1}{8}\frac{o^3p}{a^3} & +\frac{1}{16}\frac{o^2p^2}{a^2} \\
& +\frac{5}{64}\frac{o^4}{a^4} & -\frac{1}{2}\frac{pq}{a} & -\frac{1}{32}\frac{o^4p}{a^3} \\
& +\frac{1}{2}\frac{oq}{a^2} & +\frac{1}{64}\frac{o^5}{a^4} & -\frac{1}{4}\frac{opq}{a} \\
& & +\frac{1}{8}\frac{o^2q}{a^2} & +\frac{1}{256}\frac{o^6}{a^4} \\
& & & +\frac{1}{16}\frac{o^3q}{a^2}
\end{array}$$

$$\begin{array}{lllll}
\text{hoc eſt} & -\frac{r}{a}x^3 + & s x^2 - & a t x + a^2 v \propto y^2 x; \\
& +\frac{1}{4}\frac{p^2}{a^2} & +\frac{1}{4}\frac{op^2}{a^2} & +\frac{1}{4}q^2 \\
& -\frac{3}{8}\frac{o^2p}{a^3} & -\frac{1}{8}\frac{o^3p}{a^3} & +\frac{1}{16}\frac{o^2p^2}{a^2} \\
& +\frac{5}{64}\frac{o^4}{a^4} & -\frac{1}{2}\frac{pq}{a} & -\frac{1}{32}\frac{o^4p}{a^3} \\
& +\frac{1}{2}\frac{oq}{a^2} & +\frac{1}{64}\frac{o^5}{a^4} & -\frac{1}{4}\frac{opq}{a} \\
& & +\frac{1}{8}\frac{o^2q}{a^2} & +\frac{1}{256}\frac{o^6}{a^4} \\
& & & +\frac{1}{16}\frac{o^3q}{a^2}
\end{array}$$

ſimiliter ad curvam tertii gradus.

Hinc videre licet Petrum Fermatium ingenioſiſſimum quidem Mathematicum, non rectam iniviſſe viam, cùm per duas quarti gradus curvas, æquationem hanc conſtruendam ſuſceperit, nam ſimpliciùs interſectione duarum tertii gradus, ejus conſtructio haberi poterat.

In Differt. contrà Carteſ. part. 2.

Neque eum excuſat æquatio ſeptem dimenſionum, quam per curvas tertii gradus poſtea conſtruxit; nam regula, qua utitur præfatus Author, locum habet in illis æquationibus, in quibus ignota poteſtas adfectione vacat, aut certis quibuſdam terminis afficitur, ut ex eodem Authore facilè colligi poteſt; nequaquam verò in propoſita, & ſimilibus æquationibus, ſicuti hæc noſtra methodus; Quapropter fas ſit dicere Fermatium, qui errantem in Geometria Carteſium acutiſſimè videt, in eum corrigendo etiam cecutire.

Loc. citat. part. 3.

Eodem artificio perfici poſſunt conſtructiones cæterarum æquationum ſeptem, & octo dimenſionum, dummodo ad ſeptimum gradum depreſſæ, ad octavum poſtea revocentur.

Quòd ſi pro quadrato ex latere

$$\begin{array}{llll} x^4 - \frac{1}{2}ox^3 & + \frac{1}{2}px^2 & + \frac{1}{4}opx, \\ & - \frac{1}{8}o^2 & - \frac{1}{16}o^3 \\ & & - \frac{1}{2}q \end{array}$$

fin-

ſingatur quadratum ex altero latere

$$\begin{array}{lllll} x^4 - \frac{1}{2}ox^3 & + \frac{1}{4}px^2 & + \frac{1}{4}opx & + \frac{1}{2}r, \\ & - \frac{1}{8}o^2 & - \frac{1}{16}o^3 & - \frac{1}{4}oq \\ & & - \frac{1}{2}q & + \frac{3}{16}o^2p \\ & & & - \frac{1}{8}p^2 \\ & & & - \frac{5}{128}o^4 \end{array}$$

ut deſtruantur quatuor termini propoſitæ æquationis, nimirum $ox^7 - px^6 + qx^5 - rx^4$, invenientur pro conſtructione æquationis, curvæ quatuor graduum ex una parte, & trium ex altera, hoc eſt

$$\begin{array}{lllll} x^4 - \frac{1}{2}ox^3 & + \frac{1}{2}px^2 & + \frac{1}{4}opx & + \frac{1}{2}r & \propto a^3y, \\ & - \frac{1}{8}o^2 & - \frac{2}{16}o^3 & - \frac{1}{4}oq & \\ & & - \frac{1}{2}q & + \frac{3}{16}o^2p & \\ & & & - \frac{1}{8}p^2 & \\ & & & - \frac{5}{128}o^4 & \end{array}$$

$$\text{Et}\quad \frac{a}{ſ}x^3 - tx^2 + avx + \tfrac{1}{4}ar^2 \infty ay^2,$$

$$\begin{array}{llll}
+\frac{3}{8}\frac{op^2}{a^3} & +\frac{9}{32}\frac{o^2p^2}{a^3} & +\frac{1}{4}\frac{opr}{a} & -\frac{1}{4}oqr \\
-\frac{5}{16}\frac{o^3p}{a^4} & -\frac{15}{128}\frac{po^4}{a^4} & -\frac{1}{16}\frac{o^3r}{a^2} & +\frac{3}{16}\frac{o^2pr}{a} \\
-\frac{1}{2}\frac{pq}{a^2} & -\frac{1}{2}\frac{opq}{a^2} & -\frac{1}{2}qr & +\frac{1}{16}\frac{o^2q^2}{a} \\
+\frac{7}{128}\frac{o^5}{a^5} & +\frac{7}{512}\frac{o^6}{a^5} & +\frac{1}{4}\frac{oq^2}{a} & -\frac{1}{8}p^2r \\
+\frac{3}{8}\frac{o^2q}{a^3} & +\frac{1}{8}\frac{o^3q}{a^3} & -\frac{1}{16}\frac{op^3}{a^2} & -\frac{3}{32}\frac{o^3pq}{a^2} \\
-\frac{1}{2}\frac{or}{a^2} & +\frac{1}{4}\frac{q^2}{a} & +\frac{1}{8}\frac{p^2q}{a} & +\frac{1}{16}\frac{oqp^2}{a} \\
 & +\frac{1}{2}\frac{pr}{a} & +\frac{9}{128}\frac{o^4q}{a^3} & -\frac{5}{128}\frac{ro^4}{a^2} \\
 & -\frac{1}{8}\frac{o^2r}{a^2} & -\frac{5}{16}\frac{o^2pq}{a^2} & +\frac{5}{256}\frac{qo^5}{a^3} \\
 & -\frac{1}{8}\frac{p^3}{a^2} & +\frac{7}{64}\frac{o^3p^2}{a^3} & -\frac{3}{64}\frac{o^2p^3}{a^2} \\
 & & -\frac{11}{256}\frac{o^5p}{a^3} & +\frac{1}{64}p^4 \\
 & & +\frac{5}{1024}\frac{o^7}{a^5} & -\frac{15}{1024}\frac{po^6}{a^4} \\
 & & & +\frac{23}{512}\frac{p^2o^4}{a^3} \\
 & & & +\frac{25}{16384}\frac{o^8}{a^5}
\end{array}$$

Sumpta tamen pro communi poteſtate a^6y^2; nam ſi poneretur poteſtas $a^4y^2x^2$, duæ quatuor graduum curvæ haberentur, nimirum

$$\begin{array}{lllll} x^4 - \frac{1}{2}ox^3 & + \frac{1}{2}px^2 & + \frac{1}{4}opx & + \frac{1}{2}r & \propto a^2yx, \\ & - \frac{1}{8}o^2 & - \frac{1}{16}o^3 & - \frac{1}{4}oq & \\ & & - \frac{1}{2}q & + \frac{3}{16}o^2p & \\ & & & - \frac{1}{8}p^2 & \\ & & & - \frac{5}{128}o^4 & \end{array}$$

Et $ſx^3 - atx^2 + a^2vx + \frac{1}{4}a^2r^2 \propto x^2y^2,$

$$- \frac{3}{8}\frac{op^2}{a^2} + \frac{9}{32}\frac{o^2p^2}{a^2} + \frac{1}{4}opr - \frac{1}{4}aoqr$$

$$- \frac{5}{16}\frac{o^3p}{a^3} - \frac{15}{128}\frac{po^4}{a^3} - \frac{1}{16}\frac{o^3r}{a} + \frac{3}{16}o^2pr$$

$$- \frac{1}{2}\frac{pq}{a} - \frac{1}{2}\frac{opq}{a} - \frac{1}{2}aqr + \frac{1}{16}o^2q^2$$

$$+ \frac{7}{128}\frac{o5}{a4} + \frac{7}{512}\frac{o^6}{a4} + \frac{1}{4}oq^2 - \frac{1}{8}ap^2r$$

$$+ \frac{3}{8}\frac{o^2q}{a^2} + \frac{1}{8}\frac{o^3q}{a^2} - \frac{1}{16}\frac{op^3}{a} - \frac{3}{32}\frac{o^3pq}{a}$$

$$- \frac{1}{2}\frac{or}{a} + \frac{1}{4}q^2 + \frac{1}{8}p^2q + \frac{1}{16}oqp^2$$

$$+ \frac{1}{2}pr + \frac{9}{128}\frac{o4q}{a^2} - \frac{5}{128}\frac{ro^4}{a}$$

$$- \frac{1}{8}\frac{o^2r}{a} - \frac{5}{16}\frac{o2pq}{a} + \frac{5}{256}\frac{qo5}{a^2}$$

$$- \frac{1}{8}\frac{p^3}{a} + \frac{7}{64}\frac{o3p^2}{a^2} - \frac{3}{64}\frac{o^2p^3}{a}$$

$$- \frac{11}{256}\frac{o5p}{a3} + \frac{1}{64}p^4$$

$$+ \frac{5}{1024}\frac{o7}{a4} - \frac{15}{1024}\frac{po^6}{a3}$$

$$+ \frac{23}{512}\frac{p^2o^4}{a^2}$$

$$+ \frac{25}{16384}\frac{o8}{a^4}$$

Quod indicat deſtructionem multorum terminorum æquationis, ſuperfluam aliquando eſſe ad proprium pro conſtructione locum inveniendum.

At ſi accidat, ut æquationi ita præparatæ tres illi deſint termini, quos deſtruere neceſse habemus, nulla tunc fictio quadrati requiritur. Si verò aliqui ex ipſis in ea deficiant, ita quadrati latus effingere oportet, ut qui ſuperſunt, ejus ope tollantur.

Proponatur conſtruenda æquatio $x^7 * -7x^5 * +14x^3 * -7x + q \propto 0$, quæ ortum ducit ex problemate ſectionis anguli, ſive arcûs circuli in ſeptem partes æquales.

Exaltetur ad $x^8 * -7x^6 * +14x^4 * -7x^2 + qx \propto 0$, aut $x^8 \propto * +7x^6 * -14x^4 * +7x^2 - qx$, vel $x^8 \propto * +px^6 * -rx^4 * +tx^2 - vx$, poſitis pro numero 7 litera p, pro numero 14 litera r, pro altero numero 7 litera t, & pro q ſubtenſa arcûs dati litera v.

Ad deſtruendum deinde terminum $+px^6$, formetur quadratum ex latere $x^4 - \frac{1}{2}px^2$, ſcilicet $x^8 - px^6 + \frac{1}{4}p^2x^4$, addaturq; utrique parti æquationis, portio ipſius $-px^6 + \frac{1}{4}p^2x^4$, & habebitur.

$$x^8 - px^6 + \tfrac{1}{4}p^2x^4 \propto \begin{array}{l} - \\ + \frac{1}{4}p^2 \end{array} rx^4 + tx^2 - vx.$$

Fiat demũ utrique parti æqualis poteſtas $a^4y^2x^2$, & emergent æquationes $x^8 - px^6 + \frac{1}{4}p^2x^4 \propto a^4y^2x^2$, hoc eſt $x^4 - \frac{1}{2}px^2 \propto a^2yx$, ſcilicet $x^3 - \frac{1}{2}px \propto a^2y$ ad curvam tertii gradus.

$$\text{Et} \begin{array}{l} - \\ + \frac{1}{4}p^2 \end{array} rx^4 + tx^2 - vx \propto a^4y^2x^2,$$

ideſt

idest $$\begin{array}{ll} -r & x^3 + tx - v \infty a^4y^2x, \\ +\frac{1}{4}p^2 & \end{array}$$

ſive $$\begin{array}{ll} -a^3r & x^3 + a^5tx - a^6v \infty a^4y^2x, \\ +\frac{1}{4}a^2p^2 & \end{array}$$

nimirum $$\begin{array}{ll} -\frac{r}{a} & x^3 + atx - a^2v \infty y^2x \\ +\frac{1}{4}\frac{p^2}{a^2} & \end{array}$$

ſimiliter ad curvam tertii gradus.

Sed ſi pro quadrato ex latere $x^4 - \frac{1}{2}px^2$ fingatur quadratum ex altero latere $$\begin{array}{l} x^4 - \frac{1}{2}px^2 + \frac{1}{2}r, \\ \qquad\qquad\quad\; -\frac{1}{8}p^2 \end{array}$$ cujus ope evaneſcunt duo propoſitæ æquationis termini $+px^6$, & $-rx^4$; eodem quo ſuprà modo, duæ pro eius conſtructione curvæ reperientur, verum prior quatuor graduum, & poſterior duorum, nimirum $$\begin{array}{l} x^4 - \frac{1}{2}px^2 + \frac{1}{2}r \infty a^3y, \\ \qquad\qquad\quad\; -\frac{1}{8}p^2 \end{array}$$

& $$\begin{array}{llll} \frac{t}{a} & x^2 - vx + & \frac{1}{4}r^2 & \infty y^2, \\ -\frac{1}{2}\frac{pr}{a^2} & & -\frac{1}{8}\frac{p^2r}{a} & \\ +\frac{1}{8}\frac{p^3}{a^3} & & +\frac{1}{64}\frac{p^4}{a^2} & \end{array}$$

poſita tamen a^6y^2 pro communi poteſtate; nam ſi reponatur $a^4y^2x^2$, duæ quatuor graduum curvæ reperientur, ſcilicet $$\begin{array}{l} x^4 - \frac{1}{2}px^2 + \frac{1}{2}r \infty a^2yx, \\ \qquad\qquad\quad\; -\frac{1}{8}p^2 \end{array}$$

&

$$\begin{array}{ll} \& \quad atx^2 - a^2vx & + \; a^2r^2 \propto y^2x^2, \\ -\frac{1}{2}pr & -\frac{1}{8}ap^2r \\ +\frac{1}{8}\frac{p^3}{a} & +\frac{1}{64}p^4 \end{array}$$

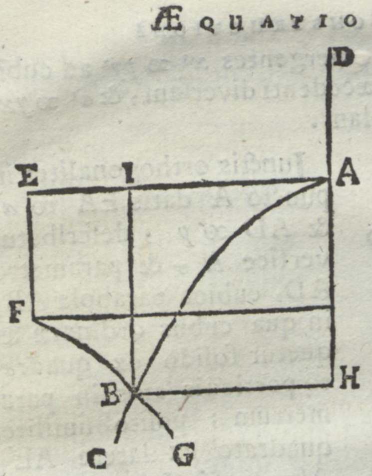
Quod ſimiliter arguit proficuam non eſſe deſtructionem duorum terminorum ex propoſita æquatione ad proprium pro conſtructione locum inveniendum.

Denique ſi proponatur æquatio $x^7 \propto a^6p$, quæ oritur, dum ſex mediæ proportionales inter datas rectas a, & p quæruntur; facilior etiam conſtructio reperietur, quia propoſita æquatio omni adfectione vacat.

Poſtquam igitur ad parem eſt elevata gradum, ſi utraque ejus pars æquetur poteſtati $a^4y^2x^2$, emergent æquationes $x^8 \propto a^4y^2x^2$, hoc eſt $x^3 \propto a^2y$ ad cubicam parabolam, & $a^6px \propto a^4y^2x^2$, nimirum $a^2p \propto y^2x$ ad cubicam hyperbolam.

Poterat autem eadem conſtructio inveniri abſque exaltatione æquationis ad parem gradum, poſita comparationis poteſtate a^4y^2x. Ut prætercam, quòd æquatione exaltata, ſi communis poteſtas aſſumeretur a^6y^2, impropria emergeret conſtructio per curvas ſecundi, & quarti gradus, quarum æquationes eſsent $px \propto y^2$, & $x^4 \propto a^3y$.

Deſcri-

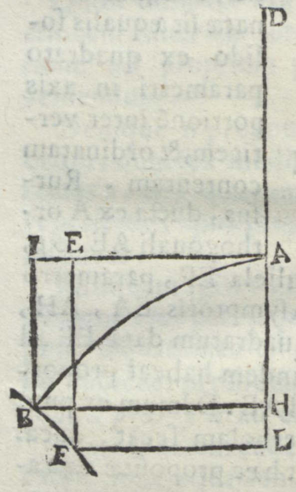


Deſcribatur, parametro AD, ſive a prima ex datis, cubica parabola ejus proprietatis, ut cubus ordinatæ ſit æqualis ſolido ex quadrato parametri in axis portionē inter verticem, & ordinatam contentam. Rurſus, ducta ex A orthogonali AE $\infty\, p$, atque ex E axi parabolæ parallela EF, parametro æquali, deſcribatur per F, Aſymptotis EA, AH, cubica hyperbola, in qua quadratum datæ EF ad quadratum ex IB, ſive AH eandem habeat proportionem, quam BH, ſive IA ad AE. Demum ex puncto B, in quo parabola hyperbolam ſecat, ducatur ordinata ad axim BH; erit hæc propoſitæ æquationis radix, nimirum prima ex quæſitis mediis, quod ita demonſtratur.

Quia BH $\infty\, x$, & per deſcriptam parabolam AH $\infty \frac{x^3}{a^2}$, ac per hyperbolam, quadratū ex AH, ſive IB, hoc eſt $\frac{x^6}{a^4}$ ad quadratum ex EF, ſive ad a^2 eandem habet rationē, quam recta AE $\infty\, p$ ad BH $\infty\, x$, erit $\frac{x^7}{a^4} \infty\, a^2p$, ſive $x^7 \infty\, a^6p$. Q. E. D.

Quod ſi cum Petro Fermatio æquetur utrique parti poteſtas py^2x^4, alia orietur constructio, ob

ob æquationes inde emergentes $x^3 \propto py^2$ ad cubicam parabolam, à præcedenti diverſam, & $a^3 \propto yx^2$ ad cubicam hyperbolam.



Junctis orthogonaliter in puncto A datis EA $\propto a$, & AD $\propto p$, deſcribatur vertice A, & parametro AD, cubica parabola AB, in qua cubus ordinatæ æquetur ſolido ex quadrato portionis axis in parametrum; poſito ſimiliter quadrato ex latere AE, Aſymptotis LA, AE, deſcribatur per F hyperbola ſecundi ordinis, in qua quadrata æquidistantium uni ex Aſymptotis, eandem habeant rationẽ, quam portiones comprehenſæ inter alteram Aſymptotum, & ipſas æquidiſtantes. Denique ex puncto interſectionis parabolæ, & hyperbolæ, ducatur ad axim ordinata BH, erit hæc radix æquationis.

Quia BH eſt $\propto x$, & DA parabolæ parameter $\propto p$, erit propter proprietatem ipſius, quadratum portionis axis AH $\propto \frac{x^3}{p}$, ac propterea latus, ſive ipſa portio AH, vel BI $\propto \sqrt{\frac{x^3}{p}}$. Rurſus quia ob deſcriptam hyperbolam, quadratum ex FL ad quadratum ex BH, hoc eſt a^2 ad x^2 eandem habet ra-

tio,

tionem, quam BI, ſive AH ad EF, vel AL, ſcilicet quam $\sqrt{\frac{x^3}{p}}$ ad a, erunt quantitates $a^4 . x^4 . \frac{x^3}{p} . a^2$ proportionales, nimirum a^4 ad x^4, ut $\frac{x^3}{p}$ ad a^2, ac proinde $\frac{x^7}{p} \infty a^6$, ſive $x^7 \infty a^6 p$. Q. E. D.

Atque hæc ſatis noſtræ methodi amplitudinem explicant: cæteras altiorum graduum æquationes eodem artificio facillimum erit conſtruere, expunctis ipſarum terminis uſque ad medietatem exponentis majoris poteſtatis, vel infrà, prout opportunius ad proprium pro conſtructione locum inveniendum exiſtimabitur; ſcilicet, ſi æquatio decem fuerit dimenſionum; ad quinque, aut quatuor, vel infrà.

Hæc tamen expunctio terminorum, duplici via perfici poteſt, vel extractione lateris quadrati ex propoſita æquatione, neglectis reſiduis, atque addito utrique æquationis parti quadrato ex eodem latere, dempto primo termino, ut altera ejus pars quadratum conſtituat, ſicuti in ſuperioribus egimus exemplis, vel efformando quadratum ex latere, quod efficiat productum ſimile propoſitæ æquationi, ut ex terminorum reſpondentium comparatione, valor illorum aſſumpti lateris exurgat, quod quidem facillimum eſt, nec longiori indiget explicatione; nam ſi ponatur, exempli gratia, æquatio ſex dimenſionum

$$x^6 - px^5 + qx^4 - rx^3 + ſx^2 - tx \infty 0;$$

ſive $x^6 \infty px^5 - qx^4 + rx^3 - ſx^2 + tx$.

Sumpto quadrato ex latere $x^3 - bx^2 + cx - d$, ſcilicet

$$\begin{array}{lllllll} x^6 & -2bx^5 & +2cx^4 & -2dx^3 & +2bdx^2 & -2dcx & +d^2, \\ & & +b^2 & -2bc & +c^2 & & \end{array}$$

ſimile hoc erit propoſitæ æquationi, proptereaque ſi termini hujus quadrati cum terminis illius comparentur, aliorum in latere aſſumptorum valor exurget, & pro latere præfato, habebimus

$$\begin{array}{llll} x^3 & -\frac{1}{2}px^2 & +\frac{1}{2}qx & -\frac{1}{2}r, \\ & & -\frac{1}{8}p^2 & +\frac{1}{4}pq \\ & & & -\frac{1}{16}p^3 \end{array}$$

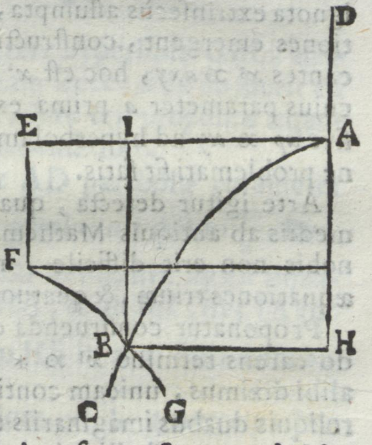
cujus ope tres æquationis termini evaneſcunt.

AD EXPLICATAS CONSTRUCTIONES ADDITAMENTUM.

Tradita eſt univerſalis Regula conſtruendi æquationes omnes, detegitur nunc Analyſis beneficio, hoc in additamento ars, qua fortaſsè Menæchmus Matematicus apud veteres celeberrimus, parabolæ, ac hyperbolæ interſectione, duas medias proportionales invenire potuerit, promoveturque hæc doctrina ad conſtructionem æquationum trium, & quatuor dimenſionum, methodo facillima, atquè elegantiſſima.

Menæchmi conſtructio ita ſe habet

Datis duabus rectis DA, & AE ad rectos angulos junctis, vertice A, deſcribatur, parabola ABC, cujus parameter AD prima ex datis, & ex termino E ductâ EF parallelâ rectę DAH, & AD æquali, ex F puncto, Aſymptotis EA, AH, deſcribatur hyperbola FBG; & Quoniam hæc hyperbola in puncto B parabolam ſecat; ſi ex eo ad axim



ducatur ordinata BH, erit hæc prior, & HA poſterior] quæſitarum mediarum, quod ita demonſtratur.

Quia propter parabolam, quadratum ex BH æquale eſt rectangulo ex DA in AH, erunt tres DA, BH, HA continuò proportionales. Similiter quia propter hyperbolam, rectangulum ex FE in EA æquale eſt rectangulo ex BH in HA, erit ut EF, ſive DA ad BH, ita HA ad AE, & quoniam per parabolam, DA, BH, HA continuò ſunt proportionales, erunt etiam DA, BH, HA, AE continuò proportionales. Q. E. D.

Ars, unde hæc conſtructio inveniri potuit, ſic Analyſis beneficio deprehenditur.

Sint datæ a, & p, atque prior ex quæſitis x; æquatio problematis naturæ conveniens, erit $x^3 \propto a^2p$.

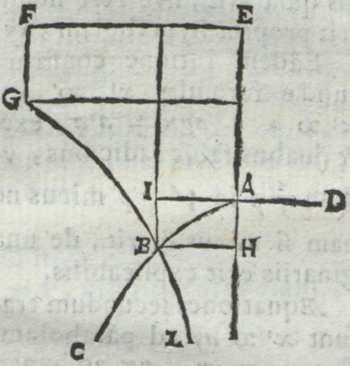
Eâ inventâ, ſi utrique parti æquetur poteſtas axy, quæ componitur ex altera datarum, quæſita, & alia ignota extrinſecùs aſſumpta, loco unius, duæ æquationes emergent, conſtructionem quæſitam indicantes $x^3 \propto axy$, hoc eſt $x^2 \propto ay$ ad parabolam, cujus parameter a prima ex datis, & $a^2p \propto axy$, ſive $ap \propto xy$ ad hyperbolam, quarum interſectione problemati fit ſatis.

Arte igitur detecta, qua problema de duabus mediis ab antiquis Mathematicis conſtrui potuit, nobis non erit difficile, methodum ad reliquas æquationes trium, & quatuor graduum extendere.

Proponatur conſtruenda cubica æquatio, ſecundo carens termino $x^3 \propto * \div px + q$, quæ, ut alibi diximus, unicam continet veram radicem, reliquis duabus imaginariis exiſtentibus.

Sumpta a, facilitatis gratia pro unitate, fiat $x^3 \propto * - apx + a^2q$, & ponatur utrique parti æqua-

æqualis poteſtas axy, emergēt æquationes $x^3 \infty axy$, ſive $x^2 \infty ay$ ad parabolam, & $-apx + a^2q \infty axy$, hoc eſt $-px + aq \infty xy$, nimirum $aq \infty xy + px$ ad hyperbolam, ex quibus facillimè conſtructio deducitur.



Deſcribatur parabola ABC, cujus parameter AD, ſive a, unitati æqualis, ſumatur deinde in axe ultrà verticem producto, portio AE æqualis p, & ducatur ex puncto E perpendicularis EF ∞q; poſtmodum, ductâ rectæ EA parallelâ FG ∞a, Aſymptotis FE, EA, per G deſcripta intelligatur hyperbola GBL, quæ cùm in B puncto parabolam ſecet, ſi ex eo fuerit ad axim ducta ordinata BH, hæc erit propoſitæ æquationis radix, quod ita demonſtratur.

Quia BH eſt ∞x, & AD parabolæ parameter ∞a, ſive unitati, erit portio axis AH $\infty \frac{x^2}{a}$. Rurſus quia rectangulum ex GF in FE, ſive aq eſt æquale rectangulis ex BH in HA, & ex BH, ſive IA in AE; ſi ducatur $\frac{x^2}{a} + p \infty$ HE in $x \infty$ BH, habebitur $\frac{x^3}{a} + px \infty aq$, ſcilicet $x^3 + apx \infty a^2p$, hoc eſt $x^3 \infty * - apx + a^2q$, vel $x^3 \infty * -px + q$. Q. E. D.

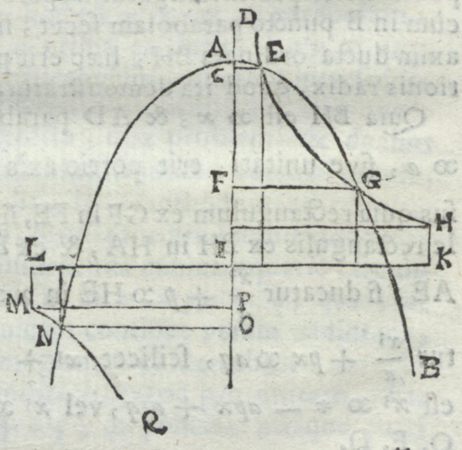
Ali-

Aliter hoc demonſtrari poteſt ex æquationibus, deductis ab illa, quam primo loco conſtruendam propoſuimus.

Quia BH eſt $\infty\ x$, & AH poſita $\infty\ y$, erit propter parabolam, cujus a parameter, $x^2 \infty\ ay$. Rurſus quia HE, ſive HA ſimul cum AE eſt $\infty\ y + p$, erit propter hyperbolam, $xy + px \infty\ aq$. Q. E. D.

Eâdem ratione conſtrui poterit æquatio ſecundæ formulæ $x^3 \infty * + px + q$, ſive $x^3 \infty * + apx + a^2q$, explicabilis de una vera, & duabus falſis radicibus, veræ æqualibus, ſi ſolidum $\frac{2}{3}p$ in $\sqrt{\frac{1}{3}p}$ minus non ſit ſolido aq, ſive q; nam ſi minus fuerit, de una vera, & duabus imaginariis erit explicabilis.

Æquationes ſecundum traditam regulam elicitæ, ſunt $x^2 \infty\ ay$ ad parabolam, & $px + aq \infty\ xy$, ſive $aq \infty\ xy - px$ ad hyperbolam, ex quibus conſtructio propoſitæ æquationis pari facilitate habetur.



Deſcribatur parabola NAB, cujus parameter AD, ſive unitas, & portio axis AI, $\infty\ p$; deinde, erectâ ex pũcto I perpendiculari ad axim IL $\infty\ q$, ducatur LM ipſi æquidiſtãs, & parabolæ parametro æqua-

lis;

lis; Poſtmodum Aſymptotis LI, IP, deſcribatur hyperbola MNQ, quæ cùm parabolam in puncto N ſecet, ſi ex eo ad axim ordinata NO ducatur, erit hæc vera radix propoſitæ æquationis.

Rurſus ducatur ex puncto I verſus alteram parabolæ partem perpendicularis ad axim IK $\infty\ q$, & ex puncto K æquidiſtans eidem axi KH parabolæ parametro æqualis; Demum Aſymptotis KI, IA, deſcribatur ex H puncto oppoſita hyperbola HGE, & quoniam hæc ſimiliter parabolam in duobus punctis E, G ſecat, ſi ex eis ad axim ordinatæ EC, GF ducantur, erunt ejuſdem æquationis falſæ radices.

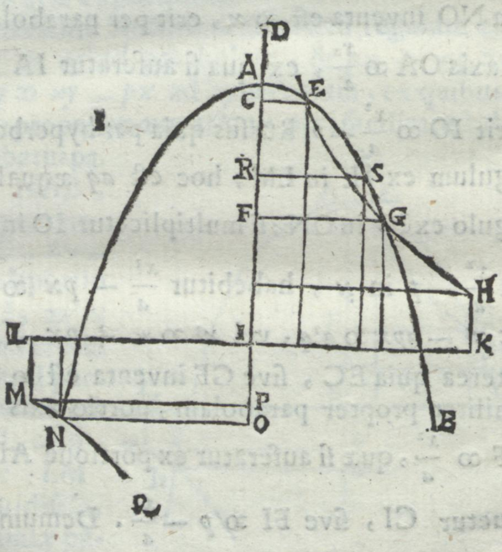
Demonſtratio ita ſe habet.

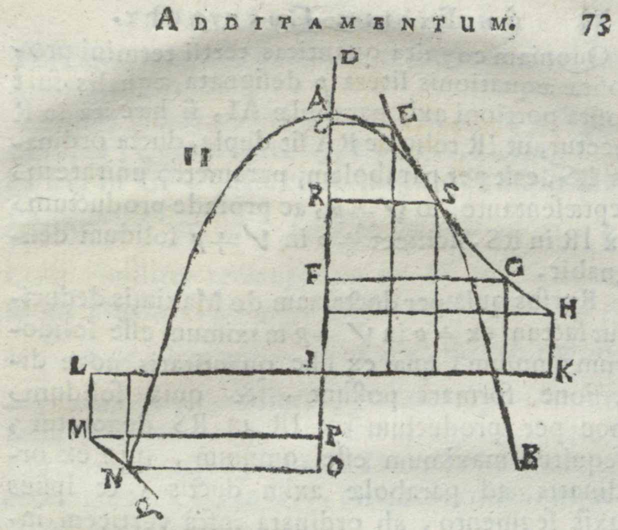
Quia NO inventa eſt $\infty\ x$, erit per parabolam portio axis OA $\infty\ \frac{x^2}{a}$, ex qua ſi auferatur IA $\infty\ p$, ſupererit IO $\infty\ \frac{x^2}{a} - p$. Rurſus quia per hyperbolam, rectangulum ex IL in LM, hoc eſt aq æquale eſt rectangulo ex IO in ON; ſi multiplicetur IO in ON, hoc eſt $\frac{x^2}{a} - p$ in x, habebitur $\frac{x^3}{a} - px\ \infty\ aq$, ſcilicet $x^3 - apx\ \infty\ a^2q$, vel $x^3\ \infty\ * + px \dotplus q$.

Præterea quia EC, ſive GF inventa eſt $\infty - x$, erit ſimiliter propter parabolam, portio axis AC, ſive AF $\infty\ \frac{x^2}{a}$, quæ ſi auferatur ex portione AI $\infty\ p$, relinquetur CI, ſive FI $\infty\ p - \frac{x^2}{a}$. Demum quia rectangulum ex HK in KI, nimirum aq eſt æquale re-

rectangulo ex EC in CI, ſive ex GF in FI; ſi ducatur $-x$ in $p - \frac{x^2}{a}$, habebitur $-px + \frac{x^3}{a} \infty aq$, hoc eſt $-apx + x^3 \infty a^2q$, nimirum $x^3 \infty * + px + q$. Q. E. D.

Explicatis jam radicibus propoſitæ æquationis; demonſtratur veritas conditionis requiſitæ, quod ſcilicet ad hoc ut propoſita æquatio unicam contineat veram radicem, & reliquas falſas, veræ æquales, oporteat ſolidum $\frac{2}{3}p$ in $\sqrt{\frac{1}{3}p}$ non minus eſſe ſolido q, aliter ſi minus fuerit, unicam veram, & reliquas imaginarias continebit.

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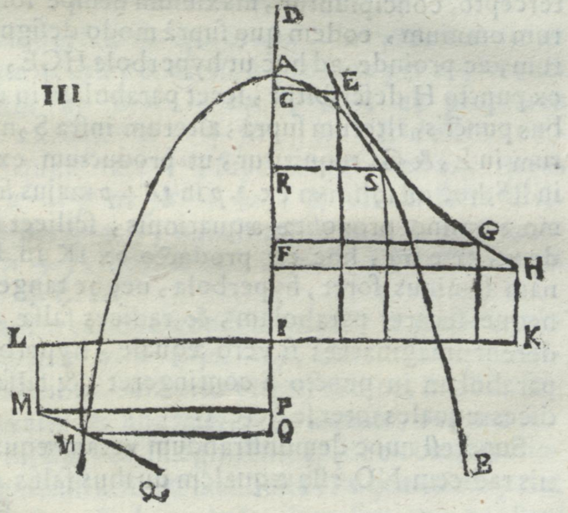
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Quoniam cognita quantitas tertii termini propoſitæ æquationis litera *p* deſignata, æqualis fuit poſita portioni axis parabolæ AI, ſi hæc ita in R ſecetur, ut IR reliquæ RA ſit dupla, ducta ordinata RS, erit per parabolam, parametro unitatem repræſentante, $\infty \sqrt{\frac{1}{3}p}$, ac proinde productum ex IR in RS, ſcilicet $\frac{2}{3}p$ in $\sqrt{\frac{1}{3}p}$ ſolidum deſignabit.

Rurſus quia per doctrinam de Maximis deducitur factum ex $\frac{2}{3}p$ in $\sqrt{\frac{1}{3}p}$ maximum eſſe ſolidorum omnium, quæ ex hac quantitatis notæ diviſione formari poſſunt, & quia ſolidum hoc per productum ex IR in RS denotatur, ſequitur maximum eſſe omnium, quæ ex ordinatis ad parabolæ axim ductis, & ipſius axis ſegmento, ab ordinata infrà verticem intercepto, concipiuntur, maximum nempè ſolidorum omnium, eodem quo ſuprà modo deſignatorum, ac proinde, ad hoc ut hyperbola HGE, quæ ex puncto H deſcribitur, ſecet parabolam in duobus punctis, alterum ſuprà, alterum infra S, nimirum in E, & G, requiritur, *Fig. I.* ut productum ex IR in RS, hoc eſt ſolidum ex $\frac{2}{3}p$ in $\sqrt{\frac{1}{3}p}$ majus ſit ultimo termino propoſitæ æquationis, ſcilicet ſolido *q*, ſive *aq*, hoc eſt producto ex IK in KH; nam ſi minus foret, hyperbola, neque tangeret, *Fig. III.* neque ſecaret parabolam, & radices falſæ evaderent imaginariæ; ſi verò æquale, hyperbola *Fig. II.* parabolam in puncto S contingeret, & falſæ radices æquales inter ſe fierent.

Supereſt nunc demonſtrandum veram æquationis radicem NO eſſe æqualem duabus falſis CE,

FG ſimul ſumptis, quod, & defectus ſecundi termini ſatis indicat.

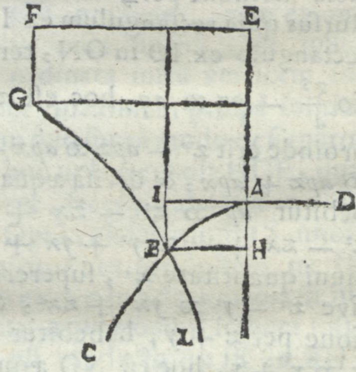
Poſitis CE $\infty\, x$, FG $\infty\, y$, NO $\infty\, z$, & reliquis, ut ſuprà, Quia propter hyperbolam, rectangulum ex EC in CI æquale eſt rectangulo ex IK in KH, erit $px - \frac{x^3}{a} \infty\, aq$, ſcilicet $apx - x^3 \infty\, a^2q$. Quia ſimiliter rectangulum ex FI in FG æquale eſt eidem rectangulo ex IK in KH, hoc eſt $py - \frac{y^3}{a} \infty\, aq$, ſive $apy - y^3 \infty\, a^2q$, habebitur $apy - y^3 \infty\, apx - x^3$, ſcilicet $apy - apx \infty\, y^3 - x^3$, & facta diviſione per $y - x$, fiet $ap \infty\, y^2 + yx + x^2$. Rurſus quia rectangulum ex IP in PM æquale eſt rectangulo ex IO in ON, tertia invenietur æquatio $\frac{z^3}{a} - pz \infty\, aq$, hoc eſt $z^3 - apz \infty\, a^2q$, & proinde erit $z^3 - apz \infty\, apx - x^3$, ideſt $z^3 + x^3 \infty\, apz + apx$, & diviſa æquatione per $z + x$, habebitur $ap \infty\, z^2 - zx + x^2$, ac propterea $z^2 - zx + x^2 \infty\, y^2 + yx + x^2$, & deleta communi quantitate x^2, ſupererit $z^2 - zx \infty\, y^2 + yx$, ſive $z^2 - y^2 \infty\, yx + zx$, & iterum facta diviſione per $z + y$, habebitur $x \infty\, z - y$, ſcilicet $z \infty\, x + y$, hoc eſt NO æqualis EC inſimul cum GF. Q. E. D. Fig. I.

Eadem ratione conſtrui poterit æquatio tertiæ formulæ $x^3 \infty * + px - q$, de tribus explicabilis radicibus, converſo tamen ordine, duabus ſcilicet veris, & una falſa, veris æquali, vel una falſa, & duabus imaginariis, atque ex conſtructione demonſtratio eſt manifeſta, ex qua etiam patet parabolæ, & hyperbolæ interſectione conſtrui

problema de anguli, ſive arcûs circuli ſectione in tres partes æquales, poſito circuli ſemidiametro $\propto a$, ſive unitati pro parabolæ parametro, BI axis portione $\propto p$, ſubtenſa dati arcûs $\propto q$, ſive IK, & quæſiti $\propto x$.

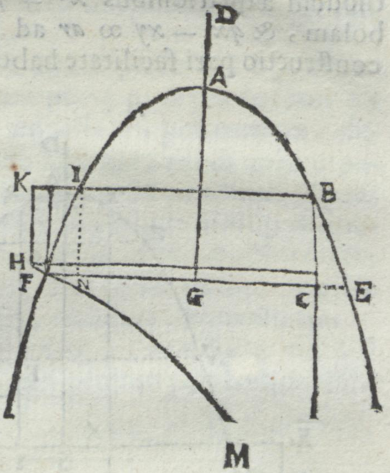
Eadem ſimiliter ratione conſtruetur æquatio $x^3 \propto * - px - q$, de unica tantùm falſa radice explicabilis.

Inventis enim duabus æquationibus $x^2 \propto ay$ ad parabolam, & $-px - aq \propto xy$, hoc eſt $-px - xy \propto aq$ ad hyperbolam, ſi deſcribatur pro conſtructione parabola ABC, cujus parameter a ſive unitas, & axis ſuprà verticem ad E producatur, ita ut AE ſit $\propto p$, poſtea ducta EF ad AE perpendiculari, ſi agatur ex F puncto recta FG $\propto a$ parallela axi, atque, Aſymptotis FE, EA, ex puncto G hyperbola GBL deſcribatur, erit ordinata BH ducta ex pũcto interſectionis parabolæ, & hyperbolæ, falſa radix propoſitæ æquationis.



Demum ſi conſtruenda proponatur æquatio $x^3 \propto px^2 * + r$, de unica tantùm vera radice explicabilis, expoſita methodo invenientur æquationes $x^2 - px \propto ay$ ad parabolam, & $ar \propto xy$ ad hyperbolam, ex quibus conſtructio facillimè deducitur.

De-



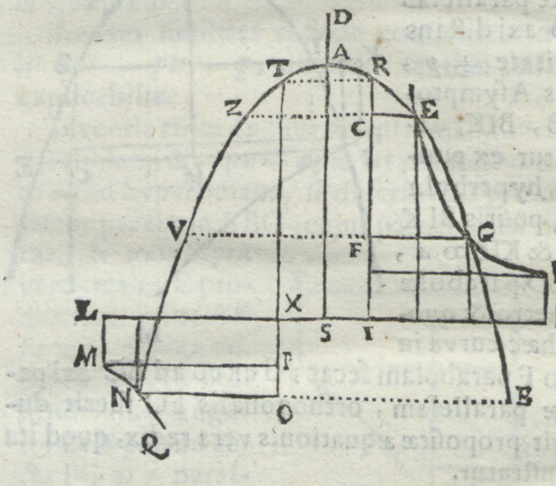
Deſcribatur parabola FAE, cujus parameter AD, ſive unitas $\propto a$, axis AG, & parallela BC ab axi diſtans quantitate $\frac{1}{2}p$; Rurſus, Aſymptotis CB, BIK, deſcribatur ex puncto H, hyperbola HFM, poſitis BIK $\propto r$, & KH $\propto a$, ſive AD parabolæ parametro, & quoniam hæc curva in puncto F parabolam ſecat; ſi ex eo ad BC axi parabolæ parallelam, orthogonalis FC fuerit ducta, erit propoſitæ æquationis vera radix, quod ita demonſtratur.

Quia FC eſt $\propto x$, & NC, ſive IB poſita $\propto p$, erit FN, ſive CE $\propto x - p$; Rurſus quia propter parabolæ proprietatem apud Pappum demonſtratam, eſt $\frac{x^2 - px}{a} \propto$ BC, habebitur per hyperbolam $\frac{x^3 - px^2}{a} \propto ar$, hoc eſt $x^3 - px^2 \propto a^2r$, nimirum $x^3 \propto px^2 * + r$. Q. E. D. *In fin. lib. 4.*

Præterea ſi proponatur cubica æquatio, in qua nullus deficit terminus, videlicet $x^3 \propto -px^2 + qx - r$, de tribus explicabilis radicibus; majori falſa, & reliquis veris, inventis juxtà me-

tho-

thodum æquationibus $x^2 + px \infty ay$ ad parabolam, & $qx - xy \infty ar$ ad hyperbolam, ejus conſtructio pari facilitate habebitur.



Deſcribatur parabola NAB, cujus axis AS, & parameter AD, ſive *a* unitati æqualis, ducaturque ex puncto R diſtante ab axi per $\frac{1}{2}p$, recta RI axi parallela, & æqualis *q*, producatur poſtea SI uſque ad K, ita ut IK ſit æqualis *r*, & ducta ex puncto K, recta KH ∞ *a*, diametro, vel axi parallela, aſymptotis RI, IK, ex puncto H deſcribatur hyperbola HGE, quæ parabolam in duobus punctis G, E ſecare poterit, ex quibus, ſi ad RI axi parallelam, fuerint ductæ orthogonales GF, EC, erunt propoſitæ æquationis veræ radices, quæ ſi æquales inter ſe fuerint, deſcripta hyperbola

para-

parabolam tanget, ſi verò imaginariæ, neque tanget, neque ſecabit.

Rurſus ducatur ex altera parte TX æqualis q, diſtans ſimiliter ab axe per $\frac{1}{2}p$, & producatur SX in L, ita ut XL ſit æqualis r, poſtmodum, ducta ex puncto L recta LM axi, vel diametro parallela $\infty\, a$, deſcribatur per M punctum, Aſymptotis LX, XO, oppoſita hyperbola MNQ, & quoniam parabola ab hac curva in puncto N ſecatur, ſi ex eo orthogonalis NO fuerit ducta, erit illa æquationis falſa radix, quod ita demonſtratur.

Quia EC, vel GF eſt $\infty\, x$, erit ZC, vel VF $\infty\, x + p$; ſimiliter quia propter parabolam, RC, vel RF eſt $\infty\, \frac{x^2 + px}{a}$, ſi hæc ab RI auferatur, ſupererit portio CI, vel FI $\infty\, q - \frac{x^2 - px}{a}$.

Demum quia propter hyperbolam, rectangulum ex EC in CI, vel ex GF in FI æquale eſt rectangulo ex IK in KH, erit $qx - \frac{x^3 - px^2}{a} \infty\, ar$, ſcilicet $aqx - x^3 - px^2 \infty\, a^2r$, hoc eſt $x^3 \infty -px^2 + qx - r$.

Rurſus quia NO eſt $\infty -x$, & rectangulum ex parabolæ parametro, & TO axi parallela, hoc eſt ay eſt æquale rectangulo ex NO in OB, ſcilicet $px + x^2$, erit OB $\infty -p - x$, & TO, ſive $y \infty\, \frac{px + x^2}{a}$; ſi igitur ex TO auferatur TX $\infty\, q$, quæ ſupereſt portio XO, erit $\infty\, \frac{px + x^2}{a} - q$, ac propterea per hyperbolam, rectangulum ex NO in

in OX æquale rectangulo ex ML in LX, ideſt $-\frac{px^2 - x^3}{a} + qx \propto ar$, ſcilicet $-px^2 - x + aqx \propto a^2r$, hoc eſt $x^3 \propto -px^2 + qx - r$. Q. E. D.

Atque hæc de cubicis æquationibus dicta ſufficiant, progrediamur nunc ad conſtructionem æquationum quatuor graduum.

Proponatur æquatio ſecundo carens termino $x^4 \propto * + qx^2 - rx + ſ$.

Aſſumpta a pro unitate, erit $x^4 \propto * + aqx^2 - a^2rx + a^3ſ$.

Poſita deinde æquali utrique parti poteſtate a^2y^2, habebitur $x^4 \propto a^2y^2$, hoc eſt $x^2 \propto ay$ ad parabolam, & $aqx^2 - a^2rx + a^3ſ \propto a^2y^2$, ſcilicet $\frac{q}{a}x^2 - rx + aſ \propto y^2$ ad hyperbolam.

Rurſus proponatur æquatio omnibus conſtans terminis $x^4 \propto px^3 + qx^2 - rx + ſ$.

Tranſmutetur hæc in $x^4 - px^3 \propto aqx^2 - a^2rx + a^3ſ$, poſita, ut ſuprà, a pro unitate.

Addatur deinde utrique parti quadratum ex $\frac{1}{2}px$, hoc eſt $\frac{1}{4}p^2x^2$, ut pars æquationis $x^4 - px^3$ quadratum conſtituat, eritque igitur

$$x^4 - px^3 + \tfrac{1}{4}p^2x^2 \propto \begin{array}{l} aqx^2 - a^2rx + a^3ſ. \\ + \tfrac{1}{4}p^2 \end{array}$$

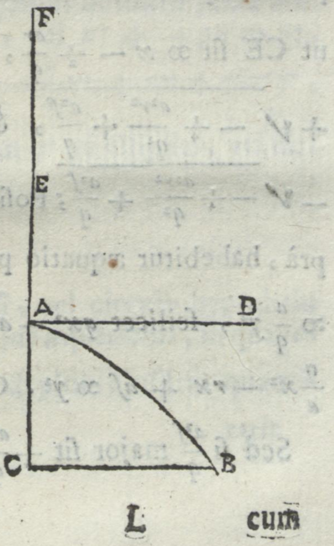
Æquetur poſtmodum utrique parti a^2y^2, & habebitur $x^4 - px^3 + \frac{1}{4}p^2x^2 \propto a^2y^2$, ſcilicet $x^2 - \frac{1}{2}px \propto ay$ ad parabolam, &

$$\begin{array}{l} aqx^2 - a^2rx \\ + \tfrac{1}{4}p^2 \end{array} + a^3ſ \propto a^2y^2, \text{ nimirum } \begin{array}{l} \frac{q}{a}x^2 - rx + aſ \propto y^2 \text{ ad} \\ + \tfrac{1}{4}\frac{p^2}{a^2} \end{array}$$

hyperbolam.

Eadem ratione erui poterunt æquationes ad curvas, conſtructioni cæterarum æquationum inſervientes, ſed notandum hyperbolam ex altera æquatione deductam, hic non deſcribi, datis aſymptotis, ſicuti in cubicis, ſed datis axe, vertice, latere recto, & tranſverſo, ut ex inventa æquatione $\frac{q}{a}x^2 - rx + aſ \infty y^2$, ſive $qx^2 - arx + a^2ſ \infty ay^2$, vel $x^2 - \frac{arx}{q} + \frac{a^2ſ}{q} \infty \frac{ay^2}{q}$ facili negotio percipi poteſt.

Si enim auferatur ſecundus terminus ex ea parte, in qua x^2 reperitur, poſita $z \infty x - \frac{1}{2}\frac{ar}{q}$, ſive $z + \frac{1}{2}\frac{ar}{q} \infty x$, habebitur æquatio $z^2 - \frac{1}{4}\frac{a^2r^2}{q^2} + \frac{a^2ſ}{q} \infty \frac{a}{q}y^2$ ad hyperbolam, cujus vertex A, tranſverſum latus AF $\infty \sqrt{-\frac{a^2r^2}{q^2} + \frac{4a^2ſ}{q}}$, hoc eſt æqualis duplo lateris ex plano $-\frac{1}{4}\frac{a^2r^2}{q^2} + \frac{a^2ſ}{q}$, rectū latus AD $\infty \sqrt{-r^2 + 4ſq}$, quartæ nimirum proportionali in ordine trium quantitatum a. q. & $\sqrt{-\frac{a^2r^2}{q^2} + \frac{4a^2ſ}{q}}$, portio axis CA unà



cum

cum dimidio tranſverſi lateris AE, ſcilicet CE $\infty\, z$, & CB ordinata $\infty\, y$, quod ita demonſtratur.

Quia CE eſt $\infty\, z$, FE, ſive EA $\infty \sqrt{-\frac{1}{4}\frac{a^2r^2}{q^2}+\frac{a^2ſ}{q}}$, erit FC $\infty\, z+\sqrt{-\frac{1}{4}\frac{a^2r^2}{q^2}+\frac{a^2ſ}{q}}$, & CA $\infty\, z-\sqrt{-\frac{1}{4}\frac{a^2r^2}{q^2}+\frac{a^2ſ}{q}}$. Rurſus quia propter hyperbolam, rectangulum ex FC in CA ſe habet ad quadratum ex CB, ſcilicet $z^2-\frac{1}{4}\frac{a^2r^2}{q^2}+\frac{a^2ſ}{q}$ ad y^2, ut $\sqrt{-\frac{a^2r^2}{q^2}+\frac{4a^2ſ}{q}}$ ad $\sqrt{-r^2+4ſq}$, hoc eſt, ut a ad q, erit $z^2-\frac{1}{4}\frac{a^2r^2}{q^2}\infty\frac{ay^2}{q}$; propterea, ſi pro z ſumatur valor ipſius $x-\frac{1}{2}\frac{ar}{q}$, ita ut CE ſit $\infty\, x-\frac{1}{2}\frac{ar}{q}$, erit FC $\infty\, x-\frac{1}{2}\frac{ar}{q}+\sqrt{-\frac{1}{4}\frac{a^2r^2}{q^2}+\frac{a^2ſ}{q}}$, & CA $\infty\, x-\frac{1}{2}\frac{ar}{q}-\sqrt{-\frac{1}{4}\frac{a^2r^2}{q^2}+\frac{a^2ſ}{q}}$; Poſitis igitur reliquis, ut ſuprà, habebitur æquatio propoſita $x^2-\frac{arx}{q}+\frac{a^2ſ}{q}\infty\frac{a}{q}y^2$, ſcilicet $qx^2-arx+a^2ſ\infty ay^2$, hoc eſt $\frac{q}{a}x^2-rx+aſ\infty y^2$. Q. E. D.

Sed ſi $\frac{a^2ſ}{q}$ major ſit $\frac{1}{4}\frac{a^2r^2}{q^2}$, ita ut quantitas

$-\frac{1}{4}$

$-\frac{1}{4}\frac{a^2r^2}{q^2}+\frac{a^2ſ}{q}$ poſt quadratum ignotum z^2, ſigno $+$ reperiatur adfecta, non diſſimili modo hyperbola manifeſtabitur, nam æquatio $z^2-\frac{1}{4}\frac{a^2r^2}{q^2}+\frac{a^2ſ}{q}\infty\frac{a}{q}y^2$, tranſpoſitionis beneficio fiet $z^2\infty\frac{a}{q}y^2+\frac{1}{4}\frac{a^2r^2}{q^2}-\frac{a^2ſ}{q}$.

At ſi tranſpoſitione uti nolumus, pari facilitate hyperbola habebitur per regulam, à Fermatio indicatam.

In ſua Iſag. ad loc. Plan. & Sol.

Notandum tamen eſt aliquando pro hyperbola ellipſim, vel circulum inveniri poſſe, quod accidere videmus, cum propoſiræ æquationis tertius terminus ſigno $-$ reperitur adfectus, ut colligi poteſt ex æquatione $x^4\infty*-qx^2+rx+ſ$, vel ex $x^4\infty-px^3-qx^2-rx+ſ$, ſive $x^4+px^3\infty-qx^2-rx+ſ$, ex quibus deductæ juxtà methodum, ſunt $x^2\infty ay$, vel $x^2+\frac{1}{2}px\infty ay$, utraque ad parabolam, & $-\frac{q}{a}x^2+rx+aſ\infty y^2$, vel $-\frac{q}{a}x^2-rx+aſ\infty y^2$ ad ellipſim, vel circulum.

$$+\frac{1}{4}\frac{p^2}{a^2}$$

Propterea ſi pro ellipſi, vel circulo hyperbola quæratur, utrique parti ejus æquationis, in qua reperitur $-\frac{q}{a}x^2$, addatur duplum ejuſdem quanti-

tatis

tatis ſigno $+$ adfectum, ad hoc ut vertatur in $+$; ſcilicet $+\frac{2q}{a}x^2$, & habebitur $+\frac{q}{a}x^2 + rx + as \infty y^2 + \frac{2q}{a}x^2$, vel $+\frac{q}{a}x^2 - rx + as \infty y^2 + \frac{2q}{a}x^2$.

$$+\tfrac{1}{4}\frac{p^2}{a^2}$$

Deinde loco x^2 ponatur ejus valor ay, vel $ay - \frac{1}{2}px$, ductoque hoc, vel illo valore, pro ut opus requirit, in $+\frac{2q}{a}$, emergent $+\frac{q}{a}x^2 + rx + as \infty y^2 + 2qy$, vel $+\frac{q}{a}x^2 - rx + as \infty y^2$

$$+\tfrac{1}{4}\frac{p^2}{a^2}$$

$+2qy - \frac{qpx}{a}$, hoc eſt $+\frac{q}{a}x^2 - rx + as \infty y^2 + 2qy$,

$$+\tfrac{1}{4}\frac{p^2}{a^2} + \frac{qp}{a}$$

utraque ad hyperbolam.

De æquationum conſtructione plura quàm initio conſtitueram congeſſi, alia addere, hominis otio abutentis munus, non injuriâ quis poſſet interpretari, ſicuti enim æquationum gradus nullis finibus coercetur, ita methodus in infinitum eſſet extendenda; quæ verò jàm ſunt explicata, ſatis, ſuperque methodi amplitudinem, & facilitatem oſtendunt, apud doctos viros commendatione non egent, illi enim probè norunt judicium, quod de his ferri debet, ab aliis autem velim noſtra hæc ſtudia, ne priùs quàm ſint intellecta, contemnantur.

FINIS.

[illegible] + ,
[illegible] $+\frac{q}{a}x^2$, & habebitur $+\frac{q}{a}x^2 +$ [illegible] y^2
$+\frac{2q}{a}x^2$, vel [illegible] $\frac{q}{a}x^2 - rx + ss \infty y^2 + \frac{2q}{a}x^2$ [illegible]

Deinde loco x^2 ponatur eius valor ay, vel $ay - \frac{1}{2}px$, ductoque eo, vel illo valore, pro ut opus requirit, in $+\frac{2q}{a}$, conveniet $+\frac{q}{a}x^2 + rx$
$+ ss \infty y^2 + 2qy$, vel $+ \frac{q}{a}x^2 - rx + ss \infty y^2$
$+ \frac{p^2}{4a}$
$+ 2qy - \frac{qpx}{a}$, [illegible] $\frac{q}{a}x^2 - rx + ss \infty y^2 + 2qy$
$+ \frac{1}{4}\frac{p^2}{a^2} + \frac{qp}{a}$
utraque ad hyperbolam.

De æquationum constructione plura quàm initio constitueram congeſſi, alia addere, hominis otio [illegible] [illegible] correctur, ita methodus in infinitum eſſet extendenda; quæ verò jam ſunt explicata, ſatis, ſuperque methodi amplitudinem, & facilitatem oſtendunt, apud doctos viros commendatione non egent, illi enim probè norunt judicium, quod de his ferri debet; ab aliis autem velim noſtra hæc ſtudia, ne priùs quàm ſint intellecta, contemnantur.

F I N I S.